

1) ohm's law

2) KVL

3) KCL

4) Thevenin's Theorem

5) Norton's Theorem

6) Maximum P.T. Theorem

7) Superposition Theorem

8) Source Transformation

9) Star-Delta Transformation

CHP-1 - DC Circuits

1) Ohm's law

→ According to Ohm's law, the potential difference across any two points of the conductor is directly proportional to the current flowing through it.

$$V \propto I$$

$$V = RI \quad [R = \text{Resistance}]$$

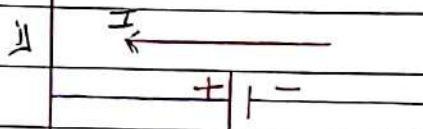
$R(\Omega) = \text{ohms}$

2) Kirchhoff's Voltage Law [KVL]

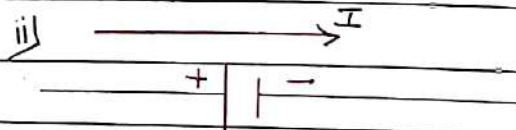
→ According to Kirchhoff's Voltage law, 'In any closed circuit or mesh, The algebraic sum of all the emf's and voltage drops will be zero.'

$$\sum \text{Emf's} + \sum IR = 0$$

#) Sign convention of KVL



Rise in Potential [+ve]



Drop in Potential [-ve]

3) Kirchhoff's Current Law [KCL]

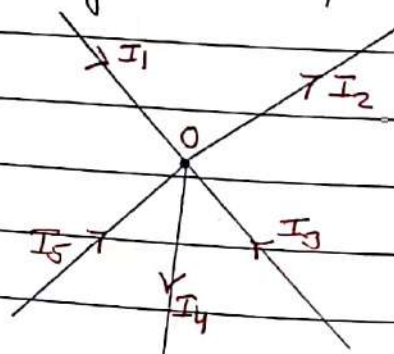
→ According to Kirchhoff's current law, The algebraic sum of all the currents meeting at a point or a junction will be zero.

$$\sum I = 0$$

#) Sign convention of KCL

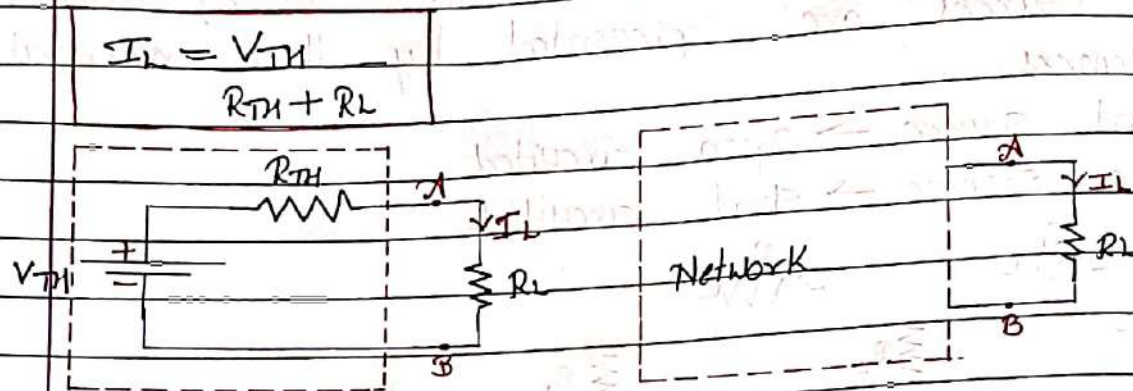
i) Incoming current → [+ve]

ii) Outgoing current → [-ve]



4) Thevenin's Theorem

→ It states that 'Any two terminals of a network can be replaced by an equivalent voltage source and an equivalent series resistance.'

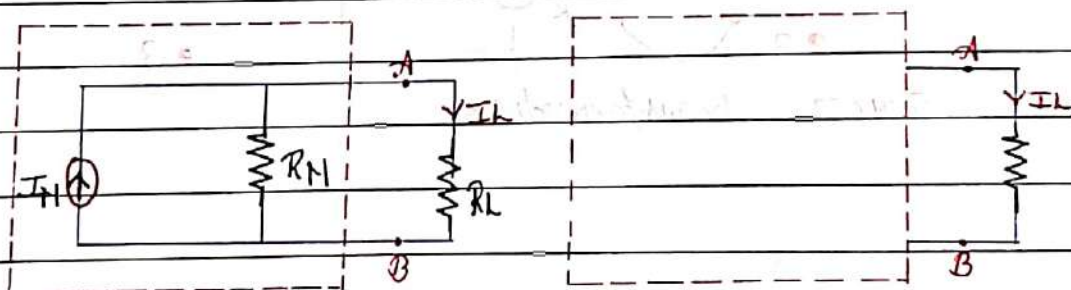


Thevenin's equivalent circuit.

5) Norton's Theorem

→ It states that 'Any two terminals of a network can be replaced by an equivalent current source and an equivalent parallel resistance.'

$$I_L = \frac{I_{N} \cdot R_N}{R_N + R_L}$$

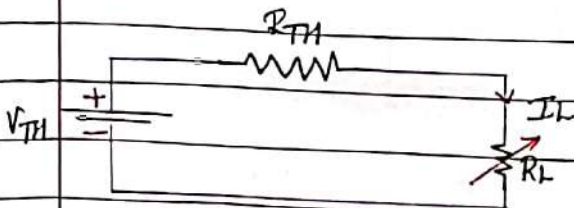


Norton's equivalent circuit.

6] Maximum Power Transfer Theorem

→ It states that 'The maximum power is delivered from a source to a load, when the load resistance is equal to the source resistance.

$$I_L = \frac{V_{TH}}{R_{TH} + R_L} \quad \text{--- ①}$$



∴ Power delivered to load R_L , is given as,

$$P = I_L^2 \cdot R_L \quad \text{--- ②}$$

$$P = \left[\frac{V_{TH}}{R_{TH} + R_L} \right]^2 \times R_L \quad \text{--- [From ①]}$$

$$P = \frac{V_{TH}^2}{(R_{TH} + R_L)^2} \cdot R_L$$

$$P = V_{TH}^2 \left[\frac{R_L}{(R_{TH} + R_L)^2} \right] \quad \text{--- ③}$$

Diff w.r.t. R_L and equating it to zero.

$$\frac{dP}{dR_L} = 0$$

$$\frac{dP}{dR_L} = V_{TH}^2 \left[\frac{(R_{TH} + R_L)^2 \times (1) - 2R_L \times (R_{TH} + R_L)}{(R_{TH} + R_L)^4} \right] = 0$$

$$(R_{TH} + R_L)^2 - 2R_L (R_{TH} + R_L) = 0$$

$$(R_{TH} + R_L) [R_{TH} + R_L - 2R_L] = 0$$

$$R_{TH} - R_L = 0$$

$$\therefore R_{TH} = R_L \quad \text{--- ④}$$

This is the required condition for max power flow.

Putting the value of $R_L = R_{TH}$ in eqn ③

$$P_{max} = \frac{V_{TH}^2 \times R_{TH}}{(R_{TH} + R_{TH})^2}$$

$$= \frac{V_{TH}^2 \times R_{TH}}{(2R_{TH})^2}$$

$$= \frac{V_{TH}^2 \times R_{TH}}{4R_{TH}^2}$$

$$P_{max} = \frac{V_{TH}^2}{4R_{TH}}$$

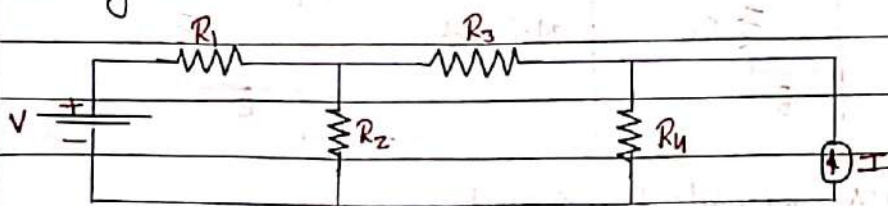
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71 Superposition Theorem

→ It states that In a linear network containing more than one independent sources, the resultant current in any element is the algebraic sum of the currents that would be produced by each source acting alone, all other sources are represented by their internal resistances.

1) Current source → Open circuited.

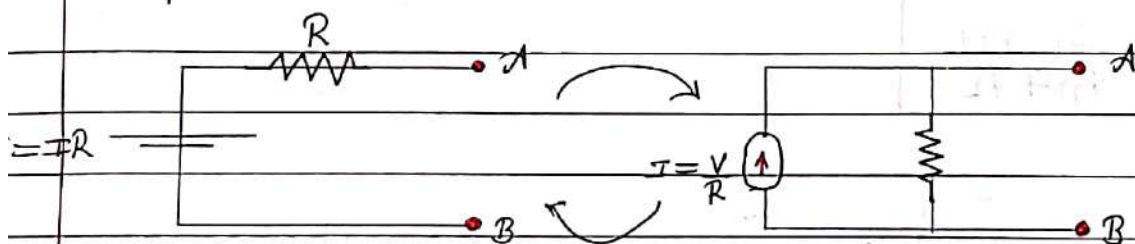
2) Voltage source → Short circuited.



Superposition theorem.

81 Source Transformation

→ A voltage source with a series resistor can be converted into an equivalent current source with a parallel resistor and vice-versa is also true.



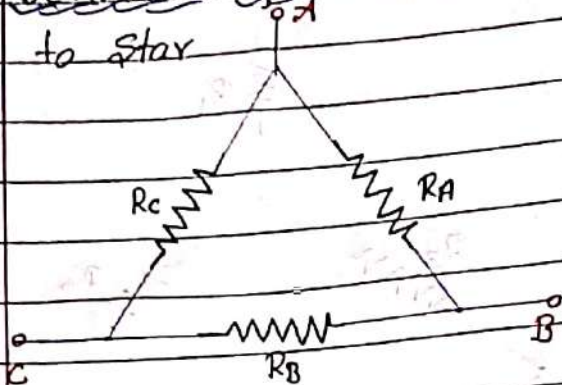
Source Transformation.

(****)

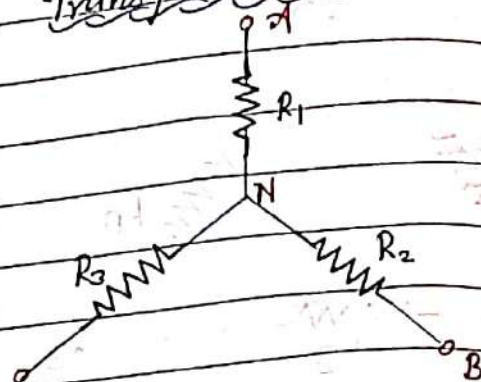
81 Derivation for

Delta-Star Transformation

i) Delta to Star



Delta



Star

ii) Equivalent Resistance between A and B

$$R_A \parallel (R_B + R_C) = \frac{R_A \cdot (R_B + R_C)}{R_A + R_B + R_C} \quad - (1)$$

Similarly,

$$R_B \parallel (R_C + R_A) = \frac{R_B \cdot (R_C + R_A)}{R_A + R_B + R_C} \quad - (2)$$

&

$$R_C \parallel (R_A + R_B) = \frac{R_C \cdot (R_A + R_B)}{R_A + R_B + R_C} \quad - (3)$$

2) Equivalent Resistance between A and B

$$R_1 + R_2 \quad - (4)$$

Similarly,

$$R_2 + R_3 \quad - (5)$$

$$R_3 + R_1 \quad - (6)$$

3) Equating ① and ④

$$R_1 + R_2 = \frac{R_A \cdot (R_B + R_C)}{R_A + R_B + R_C} \quad - (7)$$

Similarly,

$$R_2 + R_3 = \frac{R_B \cdot (R_C + R_A)}{R_A + R_B + R_C} \quad - (8)$$

&

$$R_3 + R_1 = \frac{R_C \cdot (R_A + R_B)}{R_A + R_B + R_C} \quad - (9)$$

Subtracting eqⁿ ⑧ from ⑦, we get.

$$R_1 + R_2 - R_2 - R_3 = \frac{R_A \cdot (R_B + R_C)}{R_A + R_B + R_C} - \frac{R_B \cdot (R_C + R_A)}{R_A + R_B + R_C}$$

$$R_1 - R_3 = \frac{R_A \cdot R_B + R_A \cdot R_C - R_B \cdot R_C - R_A \cdot R_B}{R_A + R_B + R_C}$$

$$R_1 - R_3 = \frac{R_A \cdot R_C - R_B \cdot R_C}{R_A + R_B + R_C} \quad - (10)$$

Adding eqⁿ ⑨ and ⑩, we get

$$R_3 + R_1 + R_1 - R_3 = \frac{R_A \cdot R_C + R_B \cdot R_C + R_A \cdot R_C - R_B \cdot R_C}{R_A + R_B + R_C}$$

$$\therefore R_1 = \frac{R_A \cdot R_C}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_A \cdot R_B}{R_A + R_B + R_C} \quad - (12)$$

and

$$R_3 = \frac{R_B \cdot R_C}{R_A + R_B + R_C} \quad - (13)$$

Similarly,

2) Star to Delta Transformation

$$R_1 = \frac{R_A \cdot R_C}{R_A + R_B + R_C} \quad \text{--- (1)} ; R_2 = \frac{R_A \cdot R_B}{R_A + R_B + R_C} \quad \text{--- (2)} ; R_3 = \frac{R_B \cdot R_C}{R_A + R_B + R_C} \quad \text{--- (3)}$$

Multiplying and adding the above eq^{ns},

$$R_1 \cdot R_2 + R_2 \cdot R_3 + R_1 \cdot R_3 = \frac{R_A^2 \cdot R_B \cdot R_C + R_A \cdot R_B^2 \cdot R_C + R_A \cdot R_B \cdot R_C^2}{(R_A + R_B + R_C)^2}$$

$$R_1 \cdot R_2 + R_2 \cdot R_3 + R_1 \cdot R_3 = \frac{R_A \cdot R_B \cdot R_C (R_A + R_B + R_C)}{(R_A + R_B + R_C)^2}$$

$$R_1 \cdot R_2 + R_2 \cdot R_3 + R_1 \cdot R_3 = \frac{R_A \cdot R_B \cdot R_C}{(R_A + R_B + R_C)^2}$$

$$R_1 \cdot R_2 + R_2 \cdot R_3 + R_1 \cdot R_3 = R_A \cdot R_3 \quad \text{--- [From (3)]}$$

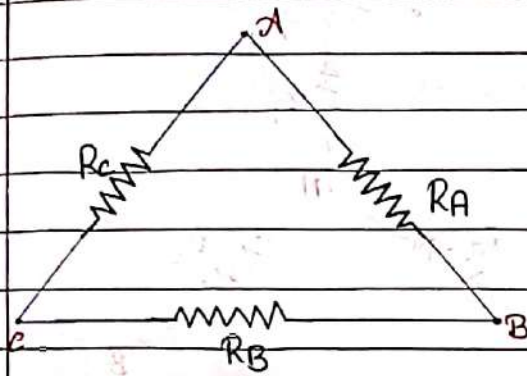
$$\therefore R_A = R_1 + R_2 + \frac{R_1 \cdot R_2}{R_3}$$

Similarly,
and

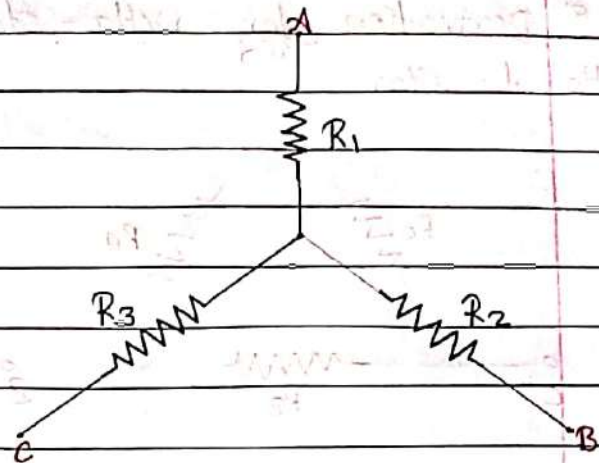
$$R_B = R_2 + R_3 + \frac{R_2 \cdot R_3}{R_1}$$

$$R_C = R_1 + R_3 + \frac{R_1 \cdot R_3}{R_2}$$

1) Star-Delta Transformation

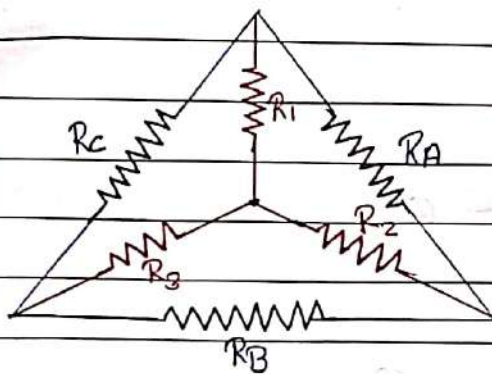


Delta Connection



Star Connection

ii) Delta to star conversion $\rightarrow$

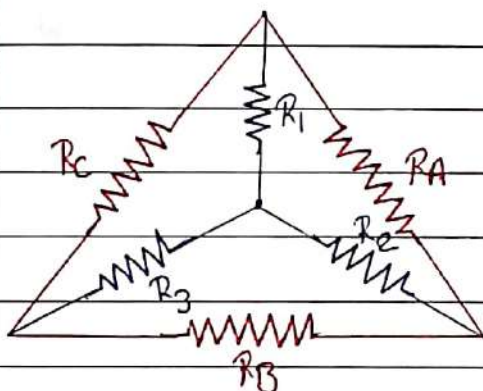


$$R_1 = \frac{R_A \cdot R_C}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_A \cdot R_B}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_B \cdot R_C}{R_A + R_B + R_C}$$

2) Star to Delta conversion $\rightarrow$



$$R_A = R_1 + R_2 + \frac{R_1 \cdot R_2}{R_3}$$

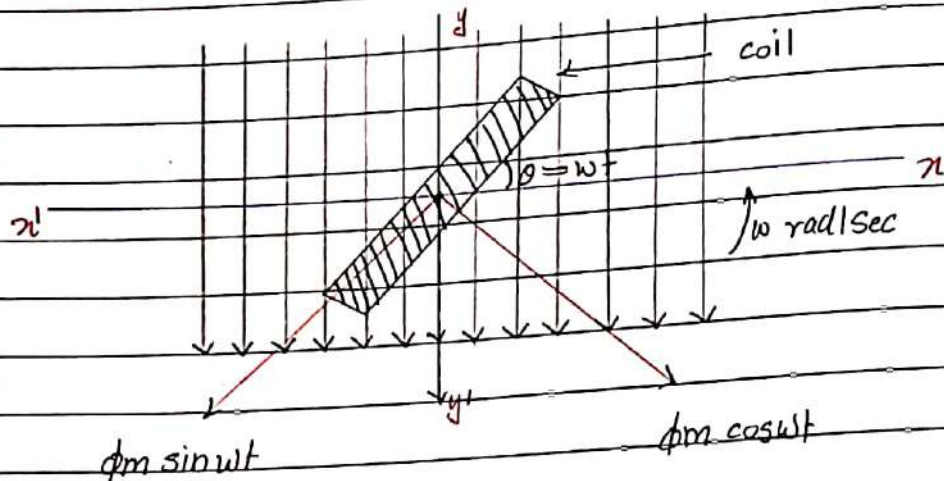
$$R_B = R_2 + R_3 + \frac{R_2 \cdot R_3}{R_1}$$

$$R_C = R_1 + R_3 + \frac{R_1 \cdot R_3}{R_2}$$

CHP-3 Ac fundamentals.

ii Generation of Alternating Voltages

→ Emf eqⁿ of single phase ac.



ii) $\phi = \phi_m \cos \omega t$ — (1)

According to Faraday's laws of EMI,

$$e = -N \frac{d\phi}{dt}$$

$$e = -N \frac{d[\phi_m \cos \omega t]}{dt} \quad \text{--- [From (1)]}$$

$$e = -N \phi_m \cdot \frac{d \cos \omega t}{dt}$$

$$e = -N \phi_m \cdot [-\sin \omega t \cdot \omega]$$

$$e = N \phi_m \cdot \omega \cdot \sin \omega t$$

$$\therefore E_m = N \phi_m \cdot \omega$$

$$i) e = E_m \cdot \sin \omega t$$

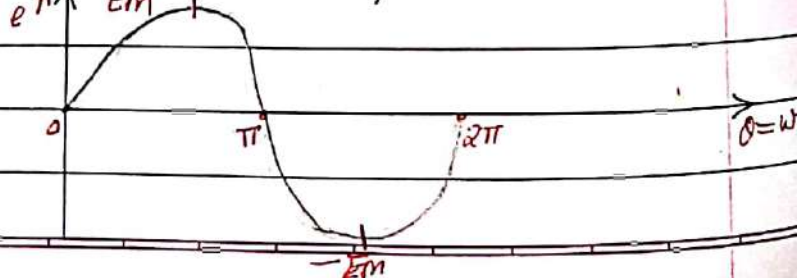
$$ii) v = V_m \cdot \sin \omega t$$

$$iii) i = I_m \cdot \sin \omega t$$

Instantaneous Value.

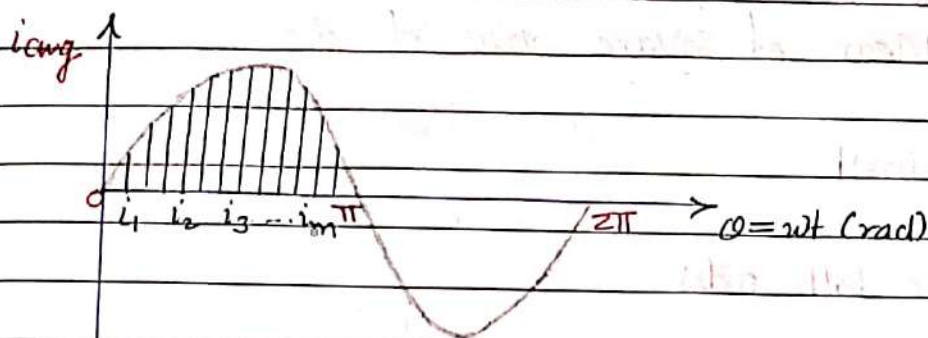
$$e = E_m \sin \omega t$$

ii) Sinusoidal waveform



[3M- Dec - 2013] - [***]

Q2) Derive an expression of Phase for the Average value of a sinusoidally varying current in terms of peak value.



Average = $\frac{\text{Sum of all values}}{\text{No. of values}}$

$$I_{avg} = \frac{i_1 + i_2 + i_3 + \dots + i_m}{m}$$

$$I_{avg} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} i \, dt$$

$$I_{avg} = \frac{1}{\pi - 0} \int_0^{\pi} I_m \sin \theta \, d\theta$$

$$= \frac{1}{\pi} \int_0^{\pi} I_m \sin \theta \, d\theta$$

$$= \frac{I_m}{\pi} \left[-\cos \theta \right]_0^{\pi}$$

$$= \frac{I_m}{\pi} [1 + 1]$$

$$I_{avg} = \frac{2I_m}{\pi}$$

$$I_{avg} = 0.6367 I_m$$

Similarly,

$$V_{avg} = 0.6367 V_m$$

Q1 Derive an expression for the RMS value of a sinusoidally varying quantity in terms of its peak value.

→ Root of Mean of Square value of AC.

$$i = I_m \sin \omega t$$

Squaring on both sides,

$$i^2 = I_m^2 \sin^2 \omega t$$

$$i_{rms}^2 = \frac{I_m^2}{\pi - 0} \int_0^{\pi} \frac{1 - \cos 2\omega t}{2}$$

$$= \frac{I_m^2}{2\pi} \left[(\omega t)_0^{\pi} - \left(\frac{\sin 2\omega t}{2} \right)_0^{\pi} \right]$$

$$= \frac{I_m^2}{2\pi} [\pi - 0]$$

$$i_{rms}^2 = \frac{I_m^2}{2}$$

$$\therefore i_{rms} = \sqrt{\frac{I_m^2}{2}}$$

$$\therefore i_{rms} = \frac{I_m}{\sqrt{2}}$$

$$\therefore i_{rms} = 0.7071 \cdot I_m$$

Similarly,

$$V_{rms} = 0.7071 \cdot V_m$$

1) What are the different Mathematical representations of Phasors?

→ A phasor can be represented in four forms.

i) Rectangular form

$$\underline{Z} = x + jy$$

$$|\underline{Z}| = \sqrt{x^2 + y^2}$$

$$\phi = \tan^{-1} \left[\frac{y}{x} \right]$$

2) Trigonometric form

$$\underline{Z} = Z (\cos \phi + j \sin \phi)$$

3) Exponential form

$$\underline{Z} = Z \cdot e^{\pm j\phi}$$

4) Polar form

$$\underline{Z} = Z \angle \pm \phi$$

5)

→ Significance of operator j → The operator j is used in rectangular form. It is used to indicate anti-clockwise rotation of a phasor through 90° .
Mathematically,

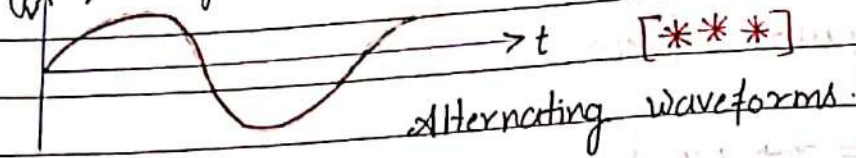
$$j = \sqrt{-1}$$

Define the following-

5) Terms Related to Alternating Quantities.

i) waveform [***]

→ A waveform is a graph in which the instantaneous value of any quantity is plotted against time.



2) Cycle.

→ One complete set of positive and negative values of an alternating quantity is termed a cycle.

3) Periodic time [***]

→ The time taken by an alternating quantity to complete one cycle is called its time period.

$$T = \frac{1}{f}$$

4) Frequency

→ The number of cycles per second of an alternating quantity is known as its frequency.

5) Amplitude

→ The maximum positive or negative value of an alternating quantity is called the Amplitude.

6) Peak factor [***]

→ It is defined as the ratio of maximum value to rms value of the given quantity.

$$\text{Peak factor } [K_p] = \frac{\text{Maximum value}}{\text{rms value}}$$

7) Form factor

→ It is defined as the ratio of rms value to the average value of the given quantity.

$$\text{Form factor } [K_f] = \frac{\text{rms value}}{\text{Average value}}$$

8) Average value [Q2M - Dec - 2016] - [***]

→ The average value of an Alternating quantity is defined as the Arithmetic mean of all the values over one complete cycle.

$$I_{\text{avg}} = \frac{i_1 + i_2 + i_3 + \dots + i_m}{m}$$

$$I_{\text{avg}} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} i(t) dt$$

q) Rms value. [02-M-DEC-2016] - [***]

→ Rms value of alternating current is defined as that value of steady current which will do the same amount of work in the same time or would produce the same heating effect as when the alternating current is applied for the same time.

$$I_{rms}^2 = \frac{i_1^2 + i_2^2 + i_3^2 + \dots + i_m^2}{m}$$

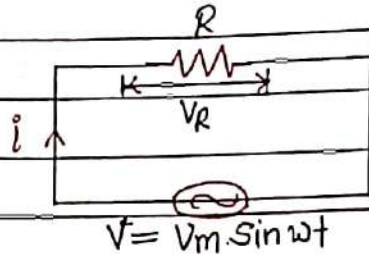
$$I_{rms} = \sqrt{\frac{i_1^2 + i_2^2 + i_3^2 + \dots + i_m^2}{m}}$$

$$I_{rms} = \sqrt{\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} i^2 dt}$$

CHP-4 Single-Phase AC Circuits

i) Behavior of a Pure Resistor in an AC circuit.

ii) Purely resistive inductive circuit.



2) Equation

3) Power factor [P.f.]

$$v = V_m \sin \omega t$$

$$P_f = \cos \phi$$

$$i = I_m \sin \omega t$$

$$P_f = \cos 0 \quad [\because \phi = 0]$$

$$P_f = 1 \quad (\text{Highest})$$

3) Impedance [Z]

4) Power

$$\rightarrow R = \frac{V}{I}$$

$$p = v i$$

$$Z = \frac{V}{i}$$

$$p = V_m \sin \omega t \times I_m \sin \omega t$$

$$p = V_m I_m \sin^2 \omega t$$

$$Z = \frac{V_m \sin \omega t}{I_m \sin \omega t}$$

$$= V_m I_m \left[\frac{1 - \cos 2\omega t}{2} \right]$$

$$Z = \frac{V_m}{I_m} = R$$

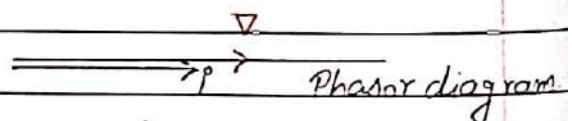
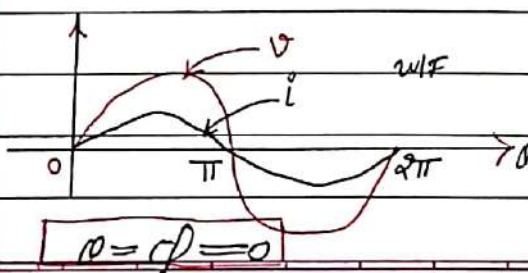
$$p = V_m I_m - V_m I_m \cos 2\omega t$$

$$Z = R$$

$$p = V_m I_m = V_m \cdot I_m \cdot \frac{\sqrt{2}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = V_{rms} \times I_{rms}$$

fluctuating power

ii) waveform / Phasor $\Rightarrow$ * Phase difference betⁿ i & v



$$\phi = 0$$

2) Behaviour of a pure inductor in an AC circuit.

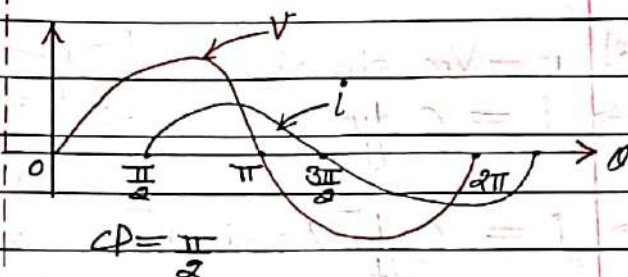
i) Purely inductive circuit

f	X_L
0	0
∞	∞



$$V = V_m \sin \omega t$$

ii) waveform



$$V = V_m \sin \omega t$$

$$i = \frac{1}{L} \int v dt$$

$$i = \frac{1}{L} \int V_m \sin \omega t dt$$

$$i = -\frac{1}{L} \cdot \frac{V_m}{\omega} \cos \omega t$$

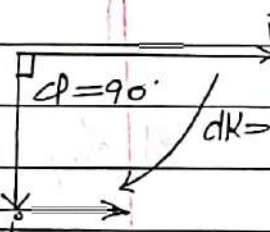
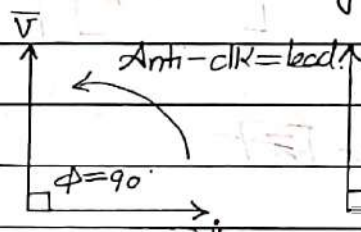
$$\left. \begin{array}{l} \omega t - \frac{\pi}{2} = 0 \\ \omega t = \frac{\pi}{2} \end{array} \right\} \phi = 0 = 90^\circ$$

$$i = -\frac{V_m}{\omega L} \cos \omega t$$

* i is lag v by $\frac{\pi}{2}$
 v is lead i by $\frac{\pi}{2}$

$$i = -\frac{V_m}{\omega L} \sin \left[\frac{\pi}{2} - \omega t \right]$$

$$i = +\frac{V_m}{\omega L} \sin \left[\omega t - \frac{\pi}{2} \right]$$



$$i = I_m \sin \left[\omega t - \frac{\pi}{2} \right]$$

$$\therefore I_m = \frac{V_m}{\omega L}$$

5) Power factor [P.f.]

$$P.f. = \cos \phi = \cos 90^\circ =$$

$$P.f. = 0$$

3) Impedance [Z]

$$Z = \frac{V_m}{I_m} = \frac{V_m}{\frac{V_m}{\omega L}} = \omega L$$

$$Z = \omega L = X_L$$

X_L = Inductive Reactance

$$X_L = \omega L = 2\pi f L \quad [\because \omega = 2\pi f]$$

$$X_L \propto f$$

6) Power

$$p = v i$$

$$p = V_m \sin \omega t \times \left[-I_m \right] \cos \omega t$$

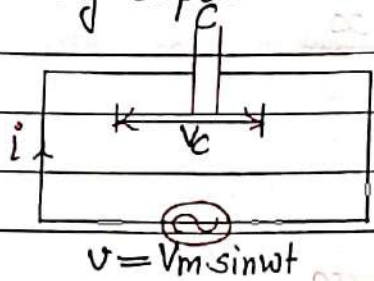
$$p = -V_m I_m \sin \omega t \cos \omega t$$

$$p = -\frac{V_m I_m}{2} \sin 2\omega t$$

$$p = 0$$

3] Behaviour of a Pure Capacitor in an AC circuit.

i] Purely capacitive circuit



f	X_c	
0	∞	$\leftarrow 0/c$
∞	0	$\leftarrow \infty/c$

ii] W/F and Phase

2] $V = V_m \sin \omega t$

$\rightarrow i = C \frac{dV}{dt}$

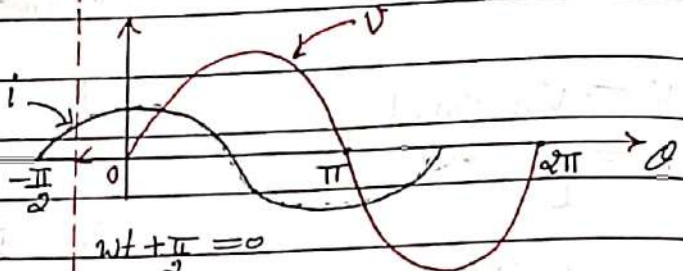
$\rightarrow i = C \frac{d[V_m \sin \omega t]}{dt}$

$i = C \cdot V_m \cdot \cos \omega t \cdot (\omega)$

$i = \omega C V_m \cos \omega t$

$i = I_m \cos \omega t$ $[\because I_m = \omega C V_m]$

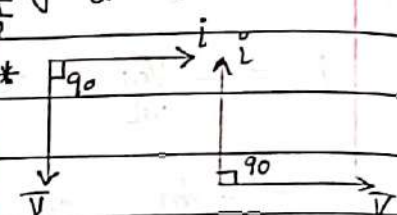
$i = I_m \sin \left[\omega t + \frac{\pi}{2} \right]$



$\omega t + \frac{\pi}{2} = 0$

$\omega t = -\frac{\pi}{2}$

* V is lag i by $\frac{\pi}{2}$
i is lead V by $\frac{\pi}{2}$
 $\phi = -\frac{\pi}{2}$



3] Impedance [Z]

$Z = \frac{V_m}{I_m} = \frac{V_{rms}}{\omega C V_{rms}} = \frac{1}{\omega C} = X_c$

$Z = X_c \rightarrow$ capacitive Reactance

$\rightarrow Z = \frac{1}{2\pi f C}$ $[\because \omega = 2\pi f]$

$Z \propto \frac{1}{f}$

5] Power factor [P.f]

$Pf = \cos \phi = \cos \left[-\frac{\pi}{2} \right] =$

$Pf = 0$

4] Power

$p = v \cdot i$

$= V_m \sin \omega t \cdot I_m \cos \omega t$

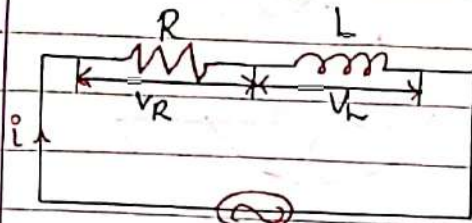
$= V_m \cdot I_m \cdot \frac{1}{2} \sin 2\omega t$

$p = \frac{V_m \cdot I_m}{2} \sin 2\omega t$

$p = 0$

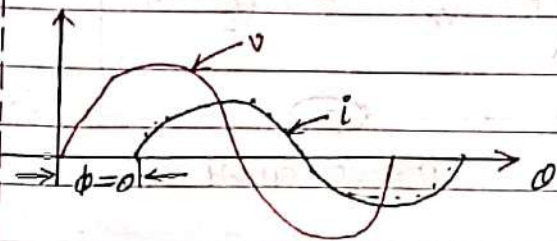
4) Behaviour of R-L series circuit.

i) Series R-L circuit.



$$v = V_m \sin \omega t$$

ii) Waveforms

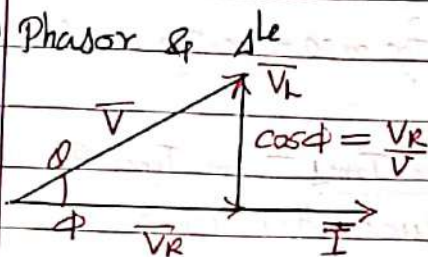


2) Equation

$$v = V_m \sin \omega t$$

$$i = I_m \sin (\omega t - \theta)$$

3) Phasor &



$$\vec{V} = \vec{V}_R + \vec{V}_L$$

$$\vec{I} = \vec{I}_R = \vec{I}_L$$

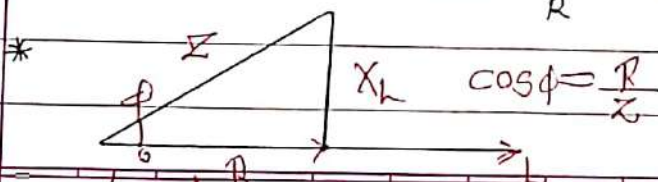
$$I_m Z = I_m R + j I_m X_L$$

$$Z = R + j X_L$$

magnitude angle

$$|Z| = \sqrt{R^2 + X_L^2} \quad \angle Z = \tan^{-1} \left[\frac{X_L}{R} \right]$$

$$|Z| \angle Z = \sqrt{R^2 + X_L^2} \angle \frac{X_L}{R}$$



Impedance Triangle

5) Power

$$p = v i$$

$$= V_m \sin \omega t \cdot I_m \sin (\omega t - \theta)$$

$$= V_m \cdot I_m [2 \sin \omega t \cdot \sin (\omega t - \theta)]$$

$$p = \frac{V_m}{2} \cdot \frac{I_m}{2} [\cos \theta - \cos (2\omega t - \theta)]$$

$$p = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos \theta - \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos (2\omega t - \theta)$$

$$p = \underline{V_{rms} \cdot I_{rms} \cos \theta} - \underline{V_{rms} \cdot I_{rms} \cos (2\omega t - \theta)}$$

fluctuating Part.

1) Power Triangle

i) Active Power [P]

$$P = VI \cos \phi \quad [\text{watt}]$$

2) Reactive Power [Q]

$$Q = VI \sin \phi \quad [\text{VAR}]$$

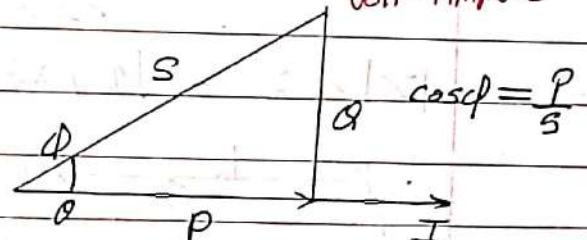
Volt-Amp-Reactive

3) Apparent Power [S]

$$S = VI$$

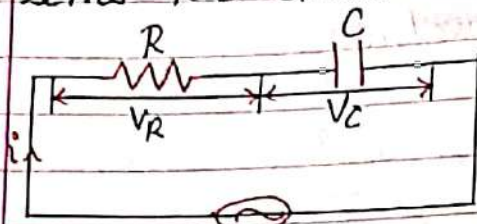
$$[\text{VA}]$$

Volt-Ampere



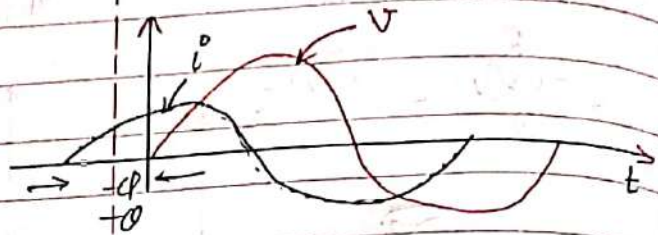
1) Behaviour of R-C series circuit.

i) Series R-C circuit.



$$v = V_m \sin \omega t$$

ii) wave-forms

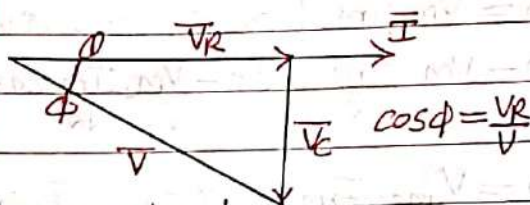


2) Equation

$$v = V_m \sin \omega t$$

$$i = I_m \sin [\omega t + \phi]$$

3) Phasor & Δ^e



Voltage triangle

$$\vec{V} = \vec{V}_R + \vec{V}_C$$

$$I_Z \quad I_R \quad I_{X_C}$$

$$I_m Z = I_m R - j I_m X_C$$

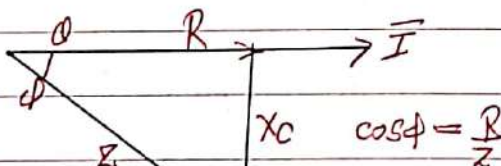
$$Z = R - j X_C$$

magnitude Angle

$$|Z| = \sqrt{R^2 + X_C^2} \quad \angle Z = \tan^{-1} \left[\frac{X_C}{R} \right]$$

$$\angle Z = -\phi, 0$$

$$|Z| \angle Z = \sqrt{R^2 + X_C^2} \angle -\frac{X_C}{R}$$



Impedance Δ^e

1) Power

$$p = v \cdot i$$

$$= V_m \sin \omega t \cdot I_m \sin (\omega t + \phi)$$

$$= V_m I_m [2 \sin \omega t \cdot \sin (\omega t + \phi)]$$

$$= V_m I_m [\cos \phi - \cos (2\omega t + \phi)]$$

$$p = \frac{V_m I_m \cos \phi}{2} - \frac{V_m I_m \cos (2\omega t + \phi)}{2}$$

$$P = \underline{V_{rms} \cdot I_{rms} - V_{rms} \cdot I_{rms} \cos (2\omega t + \phi)}$$

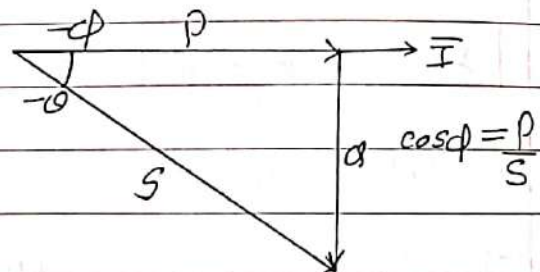
fluctuating Part.

1) Power Triangle

i) $P = VI \cos \phi$ [W]

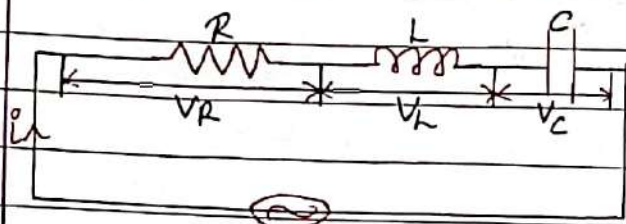
2) $Q = VI \sin \phi$ [VAR]

3) $S = VI$ [VA]

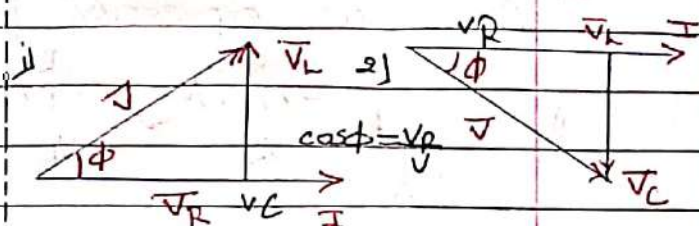


6) R-L-C Series circuit

i) Series [R-L-C] circuit.



$$V = V_m \sin \omega t$$

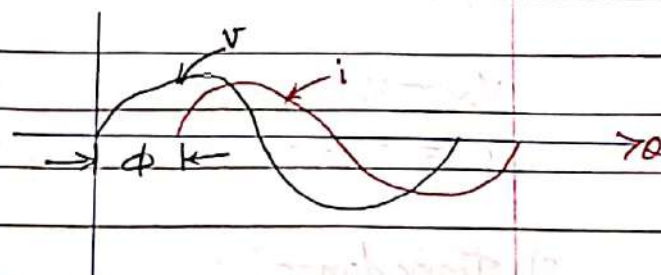


ii) wave forms

2) Equation

$$V = V_m \sin \omega t$$

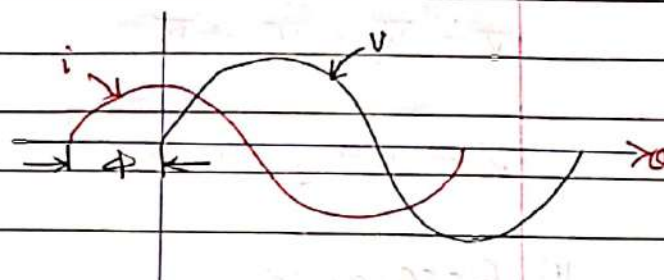
$$i = I_m \sin [\omega t \pm \phi]$$



3) Impedance

$$V = V_R + V_L + V_C$$

$$I_m Z = I_m R + j I_m X_L - j I_m X_C$$

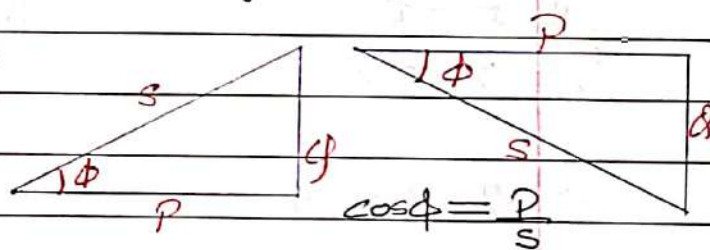


$$Z = R + j(X_L - X_C)$$

Power Triangle

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\angle Z = \tan^{-1} \left[\frac{X_L - X_C}{R} \right]$$



Power

$$P = VI \cos \phi \quad [W]$$

$$Q = VI \sin \phi \quad [VAR]$$

$$S = VI \quad [VA]$$

iv) Phasor and A.C.

$$X_L - X_C$$

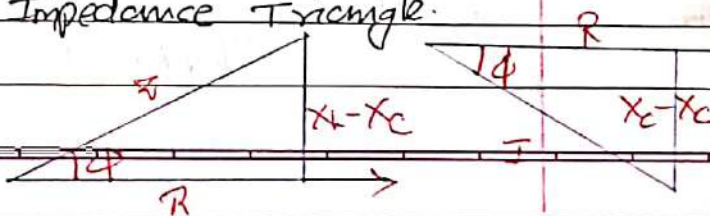
$$X_L > X_C$$

(Inductive)

$$X_L < X_C$$

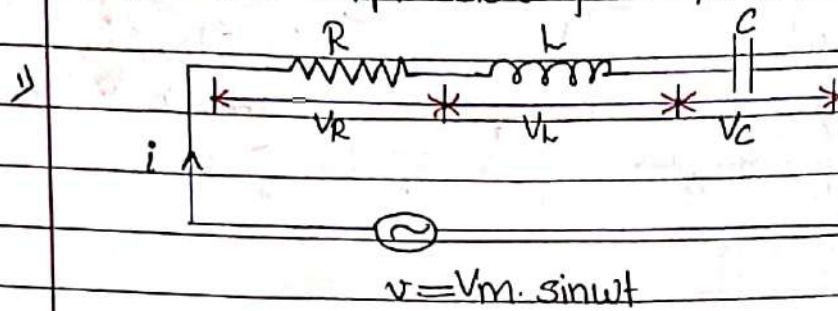
(Capacitive)

Impedance Triangle



[31-May-2016, 11-Dec-2013]

Q1 Derive an expression for Resonance frequency



2) Equation

$$V_m = V_m \sin \omega t$$

$$i = I_m \sin \omega t$$

b) Resonance,

$$Z = R$$

$$f = f_r$$

3) Impedance

$$V = V_R + V_L + V_C$$

$$Z = R + j(X_L - X_C)$$

$$\Rightarrow Z = R + j(X_L - X_C)$$

$$\text{Now, } X_L = X_C$$

$$Z = R + j(X_L - X_L)$$

$$Z = R + 0$$

$$\therefore Z = R \quad (\Omega)$$

4) Resonance

OR

$$1) Z = R$$

$$2) X_L = X_C$$

$$\omega = 2\pi f$$

$$\omega_r = 2\pi f_r$$

$$\rightarrow \text{If } X_L = X_C$$

$$\omega_r = 2\pi \times \frac{1}{2\pi \sqrt{LC}}$$

$$2\pi f_r = \frac{1}{2\pi \sqrt{LC}}$$

$$\omega_r = \frac{1}{\sqrt{LC}} \quad (\text{r/s})$$

$$f^2 = \frac{1}{4\pi^2 LC}$$

$$\therefore f_r = f = \frac{1}{2\pi \sqrt{LC}} \quad \text{Hz}$$

5)

$$Z = \frac{V}{I}$$

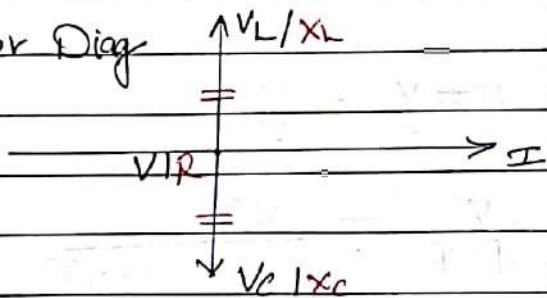
$$Z = \frac{V_m \sin \omega t}{I_m \sin \omega t}$$

$$Z = \frac{V_m}{I_m} = R$$

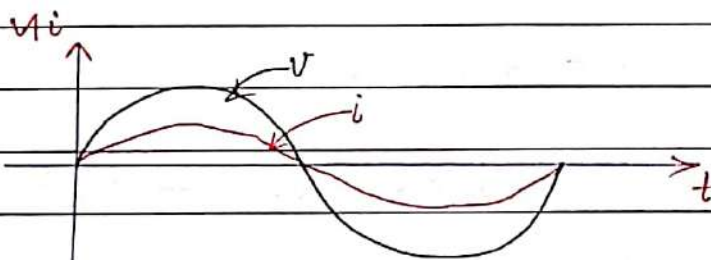
$$\therefore Z = R = \frac{\frac{V_m}{\sqrt{2}}}{\frac{I_m}{\sqrt{2}}}$$

$$\therefore Z = R = \frac{V_{rms}}{I_{rms}} = \frac{V}{I}$$

6) Phasor Diag



7) Waveforms



8) Power factor [P.f.]

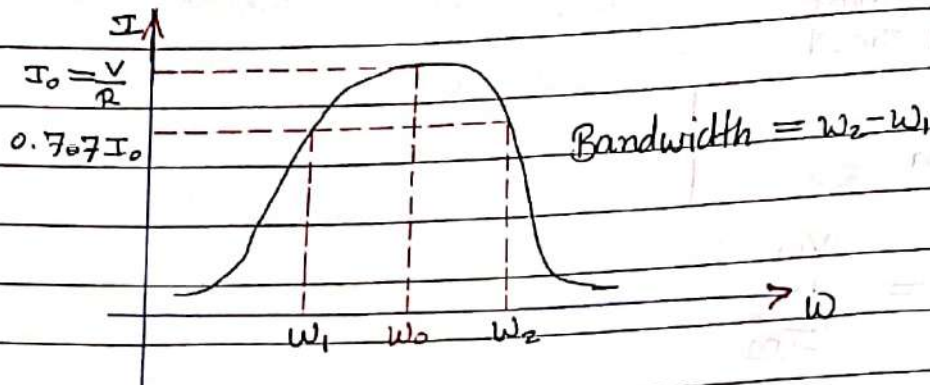
$$P.f. = \cos \phi$$

$$[\because \phi = 0]$$

$$P.f. = \cos 0 = 1$$

$$P.f. = 1$$

8] Derive the relation for Bandwidth of a series R-L-C circuit.



$$P = I^2 R$$

Half ($\frac{1}{2}$) Power Point formula

$$I_1^2 R = I_2^2 R = \frac{1}{2} I_0^2 R$$

$$I_1^2 = I_2^2 = \frac{1}{2} I_0^2$$

$$\therefore I = \frac{1}{\sqrt{2}} I_0$$

$$\therefore I_0 = \frac{V}{R}$$

$$\Rightarrow I = \frac{1}{\sqrt{2}} \cdot \frac{V}{R}$$

$$\therefore I = \frac{V}{\sqrt{2} R}$$

3] Expression for Bandwidth

$$I = \frac{V}{R} = \frac{V}{Z}$$

$$\frac{V}{\sqrt{2} R} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$\sqrt{R^2 + (X_L - X_C)^2} = \sqrt{2} R$$

Squaring on b.s.

$$\therefore R^2 + (X_L - X_C)^2 = 2 R^2$$

$$(X_L - X_C)^2 = 2 R^2 - R^2$$

$$\left[\omega L - \frac{1}{\omega C} \right]^2 = R^2$$

$$\therefore \omega L - \frac{1}{\omega C} = R$$

$$\therefore \omega L - \frac{1}{\omega C} - R = 0$$

$$\omega^2 LC - 1 - R\omega C = 0$$

$$\therefore \omega^2 LC - \cancel{R\omega C} - \frac{1}{LC} = 0$$

$$LC \left[\omega^2 - \frac{R\omega}{L} - \frac{1}{LC} \right] = 0$$

$$\therefore \omega^2 - \frac{R\omega}{L} - \frac{1}{LC} = 0 \quad *$$

$$ax^2 + bx + c = 0$$

$$a=1, \quad b=-\frac{R}{L}, \quad c=-\frac{1}{LC}$$

$$\text{Bandwidth} = \omega_2 - \omega_1 = \frac{R}{L}$$

$$\rightarrow \omega = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\text{Bandwidth} = f_2 - f_1 = \frac{R}{2\pi L}$$

$$\omega = \pm \frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} + 4 \times 1 \times \frac{1}{LC}}$$

$$\omega = \frac{R}{L} \pm \sqrt{\frac{4R^2}{4L^2} + \frac{4}{LC}}$$

$$\omega = \pm \frac{R}{2L} \pm \sqrt{\frac{R^2}{L^2} + \frac{1}{LC}}$$

small = neglected

$$\text{ii } \omega_0 = \omega + \frac{R}{2L}$$

$$2\pi f_0 = 2\pi f + \frac{R}{2L}$$

$$\div \text{by } 2\pi \quad f_0 = f + \frac{R}{4\pi L}$$

$$\therefore \omega = \pm \frac{R}{2L} \pm \sqrt{\frac{1}{LC}} = \pm \frac{R}{2L} \pm \omega_0$$

$$a) f = \frac{1}{2\pi\sqrt{LC}}$$

$$\therefore \text{ii } \omega_0 = \omega + \frac{R}{2L} \Rightarrow b) f_0 = f + \frac{R}{4\pi L}$$

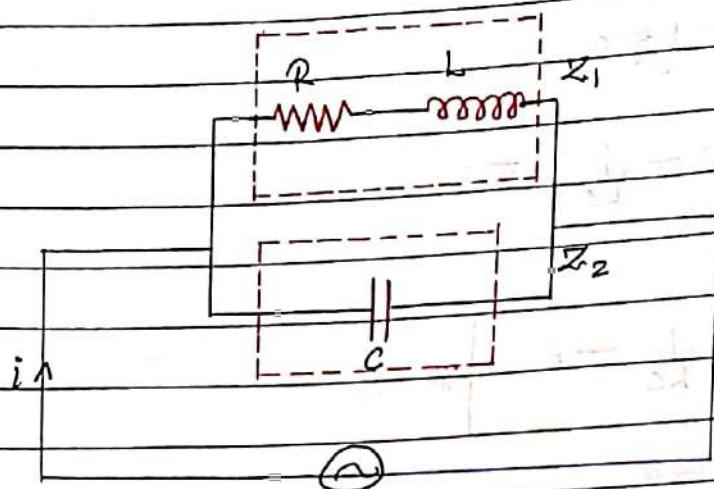
$$2\pi f = \frac{1}{\sqrt{LC}}$$

$$\text{ii } \omega_0 = \omega - \frac{R}{2L}$$

$$f_0 = f - \frac{R}{4\pi L}$$

$$\therefore \omega_0 = \frac{1}{\sqrt{LC}}$$

9) Parallel R.L.C / Parallel AC circuits



Z = Impedance

Y = Admittance

R = Resistance

$$Z_1 = R + jX_L$$

$$Z_2 = -jX_C$$

$$Y_{eq} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$1) R = \frac{1}{\text{cond}}$$

$$\text{cond} = \frac{1}{R}$$

$$Y_{eq} = \frac{1}{R + jX_L} + \frac{1}{-jX_C}$$

$$= \frac{1}{R + jX_L} + \frac{j}{X_C}$$

$$2) Z = \frac{1}{Y}$$

$$Y = \frac{1}{Z}$$

$$= \frac{(R - jX_L)}{(R + jX_L)(R - jX_L)} + \frac{j}{X_C}$$

$$3) \left. \begin{aligned} \frac{1}{Z_{eq}} &= \frac{1}{Z_1} + \frac{1}{Z_2} \\ Y_{eq} &= Y_1 + Y_2 \end{aligned} \right\} \text{Parallel}$$

$$Y_{eq} = \frac{R - jX_L}{R^2 + X_L^2} + \frac{j}{X_C}$$

$$Y_{eq} = \frac{R}{R^2 + X_L^2} - \frac{jX_L}{R^2 + X_L^2} + \frac{j}{X_C}$$

$$Y_{eq} = \frac{R}{R^2 + X_L^2} + j \left[\frac{1}{X_C} - \frac{X_L}{R^2 + X_L^2} \right]$$

$$4) \left. \begin{aligned} Z_{eq} &= Z_1 + Z_2 \\ \frac{1}{Y_{eq}} &= \frac{1}{Y_1} + \frac{1}{Y_2} \end{aligned} \right\} \text{series}$$

$$Y_{eq} = G + jB \quad \begin{matrix} (V) \text{ or } (S) \\ \text{mhos} \quad \text{Siemens} \end{matrix}$$

$$1) G = \frac{R}{R^2 + X_L^2} = \text{Conductance}$$

$$2) B = \frac{1}{X_C} - \frac{X_L}{R^2 + X_L^2} = \text{Susceptance}$$

* Note $\Rightarrow$

1) R-C Series

$$Z = R - jX_C$$

$$\frac{1}{Z} = Y = \frac{1}{R - jX_C} \times \frac{(R + jX_C)}{(R + jX_C)}$$

$$Y = \frac{R + jX_C}{R^2 + X_C^2}$$

$$Y = \frac{R}{R^2 + X_C^2} + j \frac{X_C}{R^2 + X_C^2}$$

$$\therefore Y = G + jB$$

2) R-L Series

$$Z = R + jX_L$$

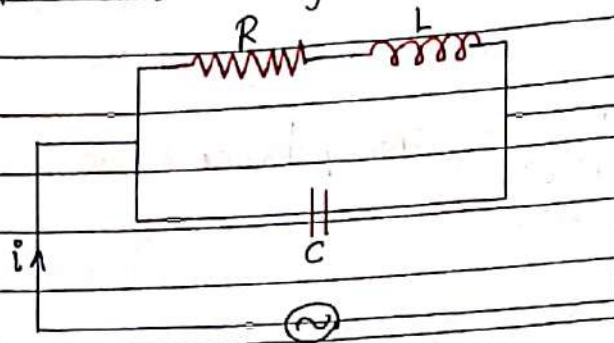
$$\frac{1}{Z} = Y = \frac{1}{R + jX_L} \times \frac{(R - jX_L)}{(R - jX_L)}$$

$$Y = \frac{R - jX_L}{R^2 + X_L^2}$$

$$Y = \frac{R}{R^2 + X_L^2} - j \frac{X_L}{R^2 + X_L^2}$$

$$\therefore Y = G - jB$$

10] Derive the condition for resonance in Parallel RLC.



$$v = V_m \sin \omega t.$$

$$Z_1 = R + jX_L$$

$$Z_2 = -jX_C$$

$$Y_{eq} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$= \frac{1}{R + jX_L} + \frac{1}{-jX_C}$$

$$= \frac{1}{R + jX_L} + \frac{j}{X_C}$$

$$= \frac{(R - jX_L)}{(R + jX_L)(R - jX_L)} + \frac{j}{X_C}$$

$$= \frac{R - jX_L}{R^2 + X_L^2} + \frac{j}{X_C}$$

$$= \frac{R}{R^2 + X_L^2} - \frac{jX_L}{R^2 + X_L^2} + \frac{j}{X_C}$$

$$= \frac{R}{R^2 + X_L^2} + j \left[\frac{1}{X_C} - \frac{X_L}{R^2 + X_L^2} \right]$$

The circuit is purely resistive.

$$\frac{1}{X_C} - \frac{X_L}{R^2 + X_L^2} = 0$$

$$\frac{1}{X_C} = \frac{X_L}{R^2 + X_L^2}$$

$$R^2 + X_L^2 = X_L \cdot X_C$$

$$\omega = \sqrt{\frac{1}{LC}}$$

$$X_L = \omega L ; X_C = \frac{1}{\omega C}$$

$$R^2 + \omega^2 L^2 = \omega L \times \frac{1}{\omega C}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$\omega = 2\pi f$$

$$2\pi f = \frac{1}{\sqrt{LC}}$$

$$R^2 + \omega^2 L^2 = \frac{L}{C}$$

$$\therefore f = \frac{1}{2\pi\sqrt{LC}}$$

$$\omega^2 L^2 = \frac{L}{C} - R^2$$

Divide by L^2

$$\frac{\omega^2 L^2}{L^2} = \frac{L}{L^2 C} - \frac{R^2}{L^2}$$

$$\omega^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\therefore \omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$\frac{\omega}{2\pi f} = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

very small

III) Define and find the expression for quality factor of a series R-L-C circuit.

→ Defination → It is a measure of voltage magnification in the series resonant circuit.

Q_s = Voltage across inductor or capacitor voltages at resonance (R)

$$= \frac{V_L}{V} = \frac{V_C}{V}$$

$$= \frac{I \cdot X_L}{I R}$$

$$= \frac{X_L}{R}$$

$$= \frac{\omega L}{R} \quad [\because X_L = \omega L]$$

$$= \frac{1}{\sqrt{LC}} \cdot \frac{L}{R} \quad [\because \omega = \frac{1}{\sqrt{LC}}]$$

$$= \frac{\cancel{L} \cdot \cancel{L}}{\cancel{L} \cdot \sqrt{C} \cdot R}$$

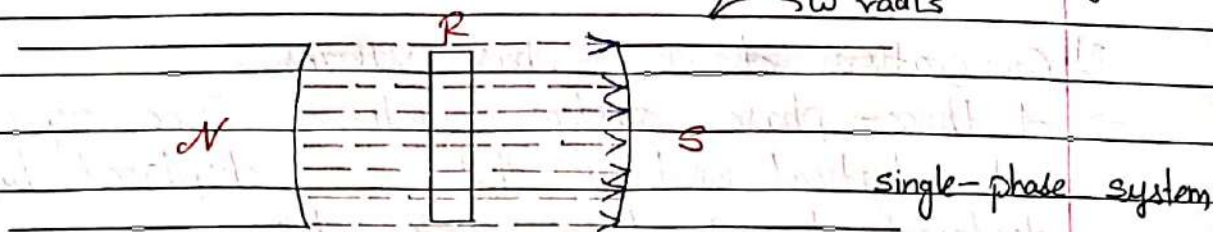
$$Q = \frac{L}{R \sqrt{C}}$$

CHP-4 Three-Phase AC

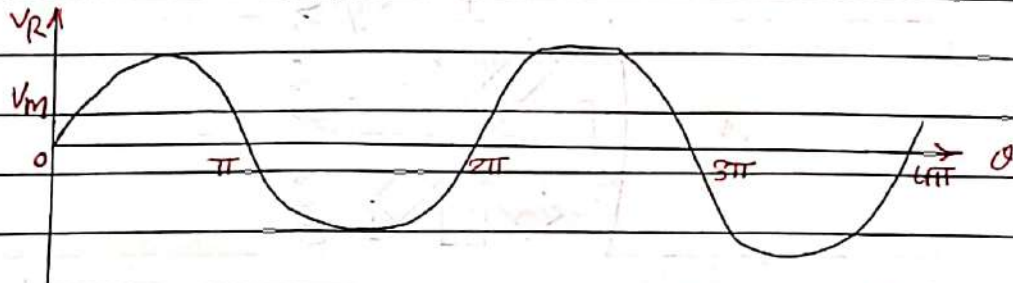
#1) Generation of three phase AC

1) Generation of single-phase voltage.

→ A single-phase system utilizes single winding.



2) waveforms

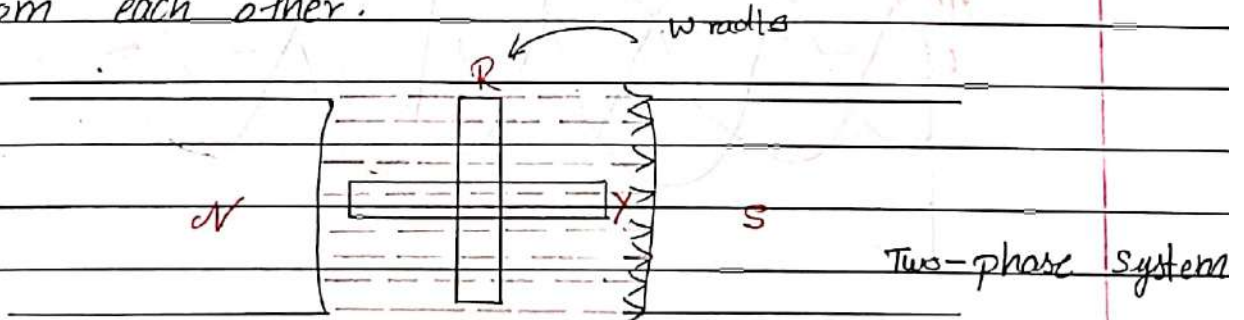


3) Phasor diagram →

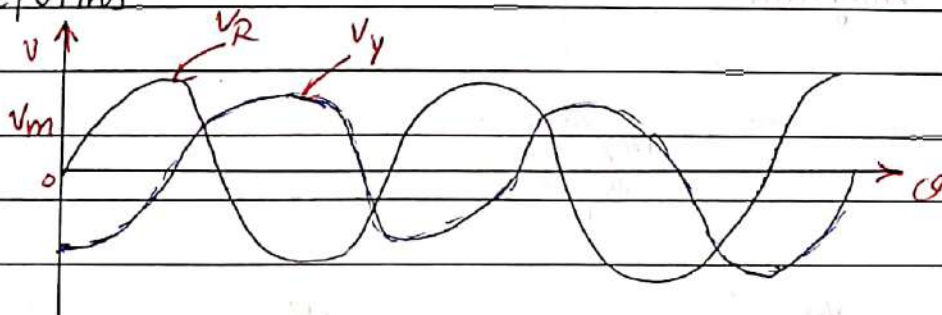
→ V_R

2) Generation of Two-phase voltages.

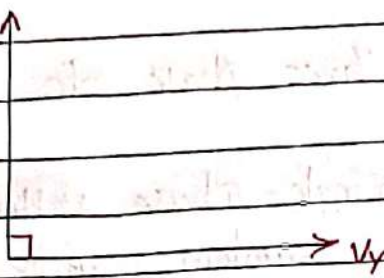
→ A two-phase system utilizes two identical windings that are displaced by 90 electrical degrees apart from each other.



2) waveforms

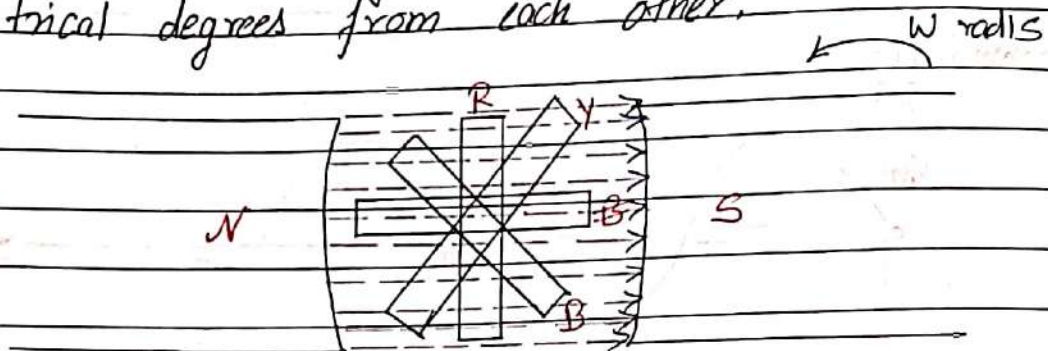


2) Phasor diagram V_R



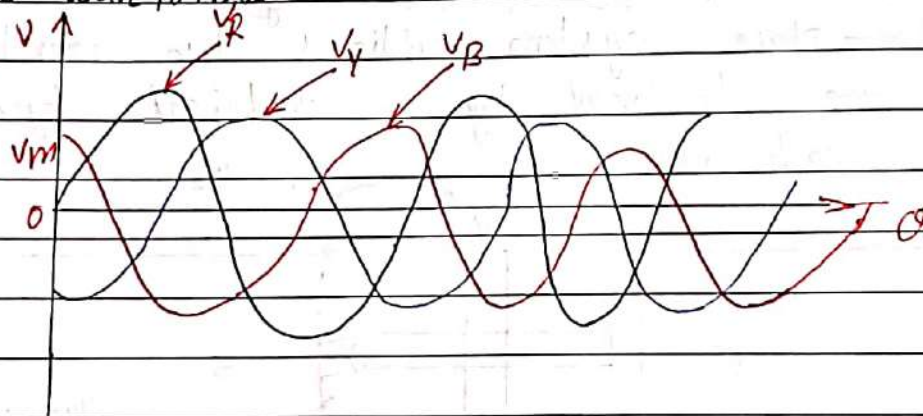
3) Generation of three-phase voltages:

→ A three-phase system utilizes three separate but identical windings that are displaced by 120 electrical degrees from each other.

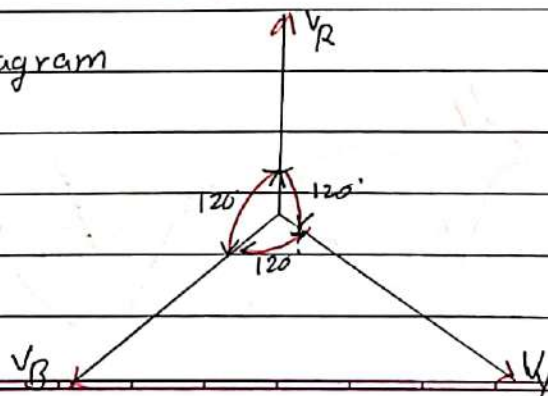


Three-phase system

4) Voltage waveforms

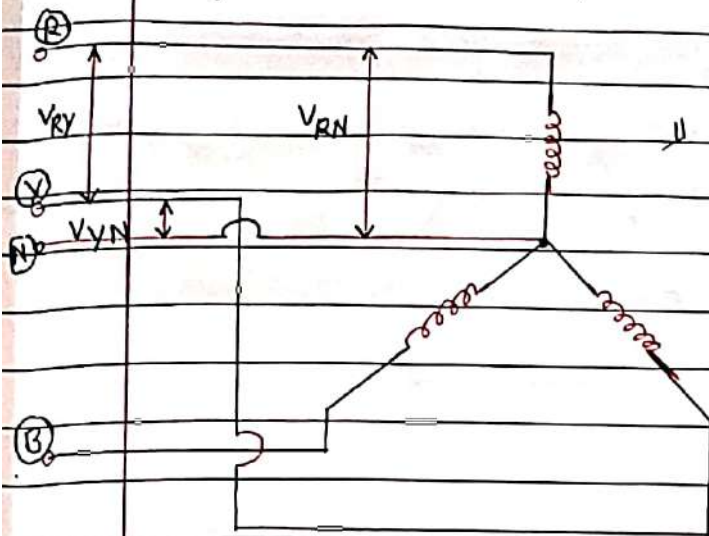


5) Phasor diagram



11 voltage and current eqⁿ in * connection. [0.5 Marks]

[Dec-2013, May-2013]



let

$$V_{RN} = V_{YN} = V_{BN} = V_{ph}$$

$$V_{RY} = V_{YB} = V_{BR} = V_L$$

$$V_{RN} = V_{ph} < 0^\circ$$

$$V_{YN} = V_{ph} < -120^\circ$$

$$V_{BN} = V_{ph} < -240^\circ$$

Applying Kirchhoff's Voltage law,

$$V_L = V_{ph1} + V_{ph2}$$

$$V_{RY} = V_{RN} + \vec{V_{YN}}$$

$$V_{RY} = V_{RN} - \vec{V_{YN}}$$

$$V_{RY} = V_{ph} < 0 - V_{ph} < -120$$

$$V_{RY} = V_{ph} [1 + j0 - (-0.5 - j0.866)]$$

$$V_{RY} = V_{ph} [1 + 0.5 + j0.866]$$

$$V_{RY} = V_{ph} [1.5 + j0.866]$$

$$V_L = V_{ph} [1.5 + j0.866]$$

$$V_L = V_{ph} [\sqrt{3} < 30^\circ]$$

$$V_L = \sqrt{3} \cdot V_{ph} < 30^\circ$$

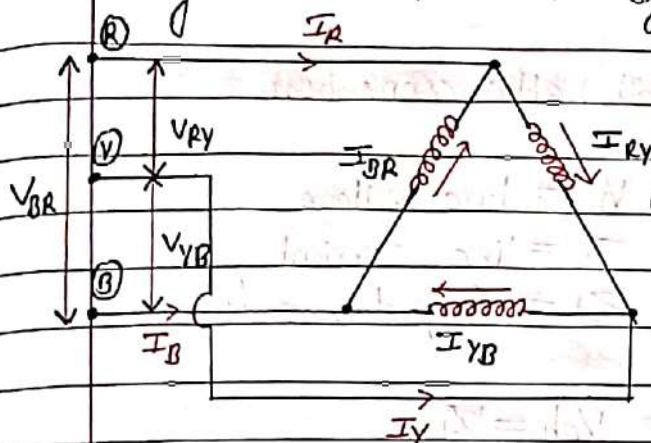
$$I_L = I_{ph}$$

Thus, in a star-connected, 3-phase system, $V_L = \sqrt{3} V_{ph}$ and line voltages lead respective phase voltages by 30° .

1) LA $P = V \cdot I \cdot \cos \phi = V_{ph} \cdot I_{ph} \cdot \cos \phi$

2) 30 $P = \sqrt{3} \cdot V_L \cdot I_L \cdot \cos \phi$ W
 $Q = \sqrt{3} \cdot V_L \cdot I_L \cdot \sin \phi$ VAR
 $S = \sqrt{3} \cdot V_L \cdot I_L$ VA

2) Voltage & current in Δ system connection. [0.5 Marks]



$$V_L = V_{ph}$$

$$① \text{ let } I_R = I_Y = I_B = I_{ph}$$

$$I_{RY} = I_{YB} = I_{BR} = I_{ph}$$

$$② \quad I_{RY} = I_{ph} \angle 0$$

$$I_{YB} = I_{ph} \angle -120$$

$$I_{BR} = I_{ph} \angle -240$$

Applying Kirchhoff's Current Law,

$$I_R + I_{BR} = I_{RY}$$

$$I_R = I_{RY} - I_{BR}$$

$$= I_{ph} \angle 0 - I_{ph} \angle -240$$

$$= I_{ph} [1 + j0 - 60.5 + j0.866]$$

$$= I_{ph} [1 + 0.5 - j0.866]$$

$$= I_{ph} [1.5 - j0.866]$$

$$I_R = I_{ph} [\sqrt{3} \angle -30^\circ]$$

$$I_L = \sqrt{3} I_{ph} \angle -30^\circ$$

Thus, in a delta connected, 3-phase system, $I_L = \sqrt{3} \cdot I_{ph}$ as line current are 30° , behind the respective phase currents.

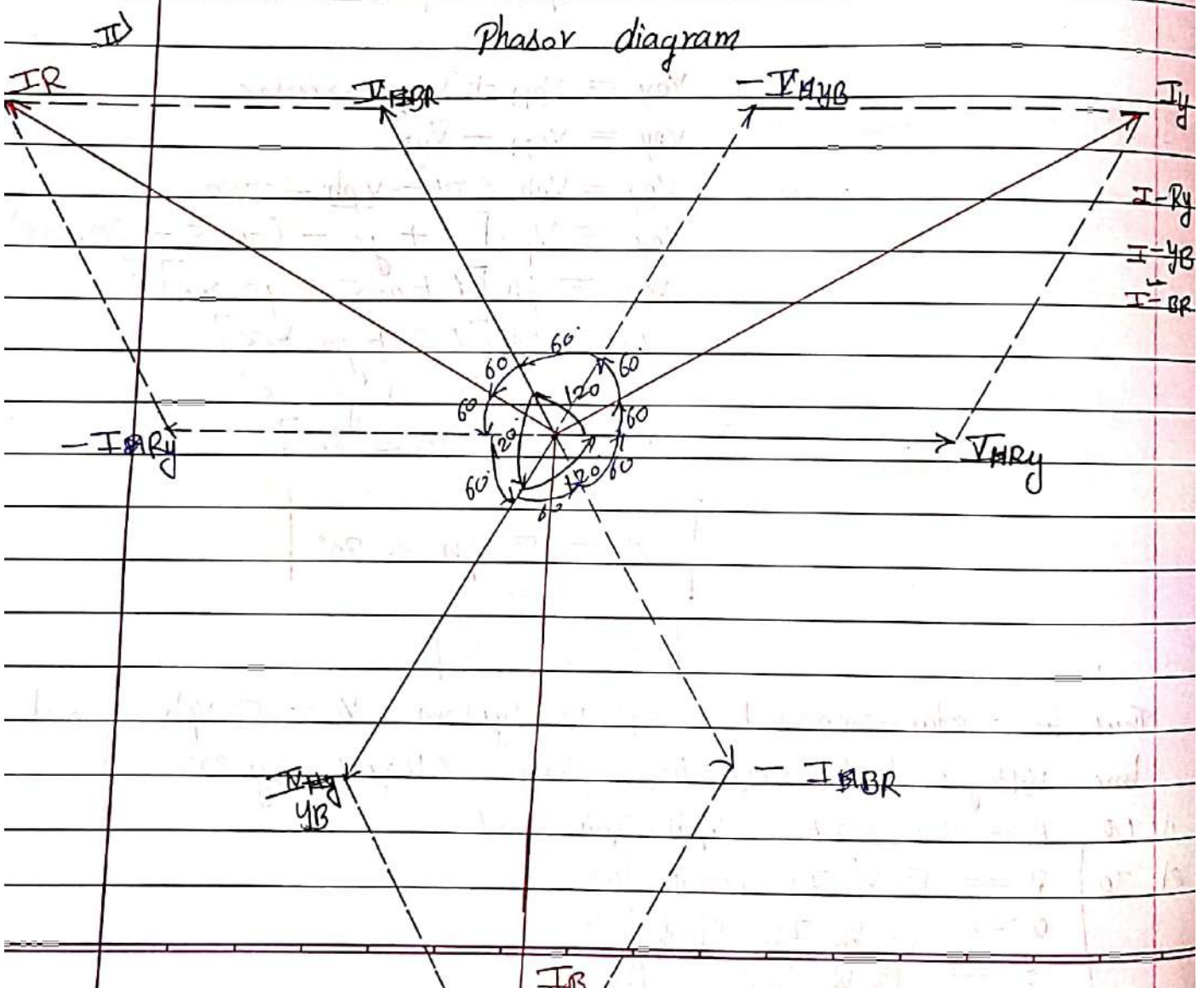
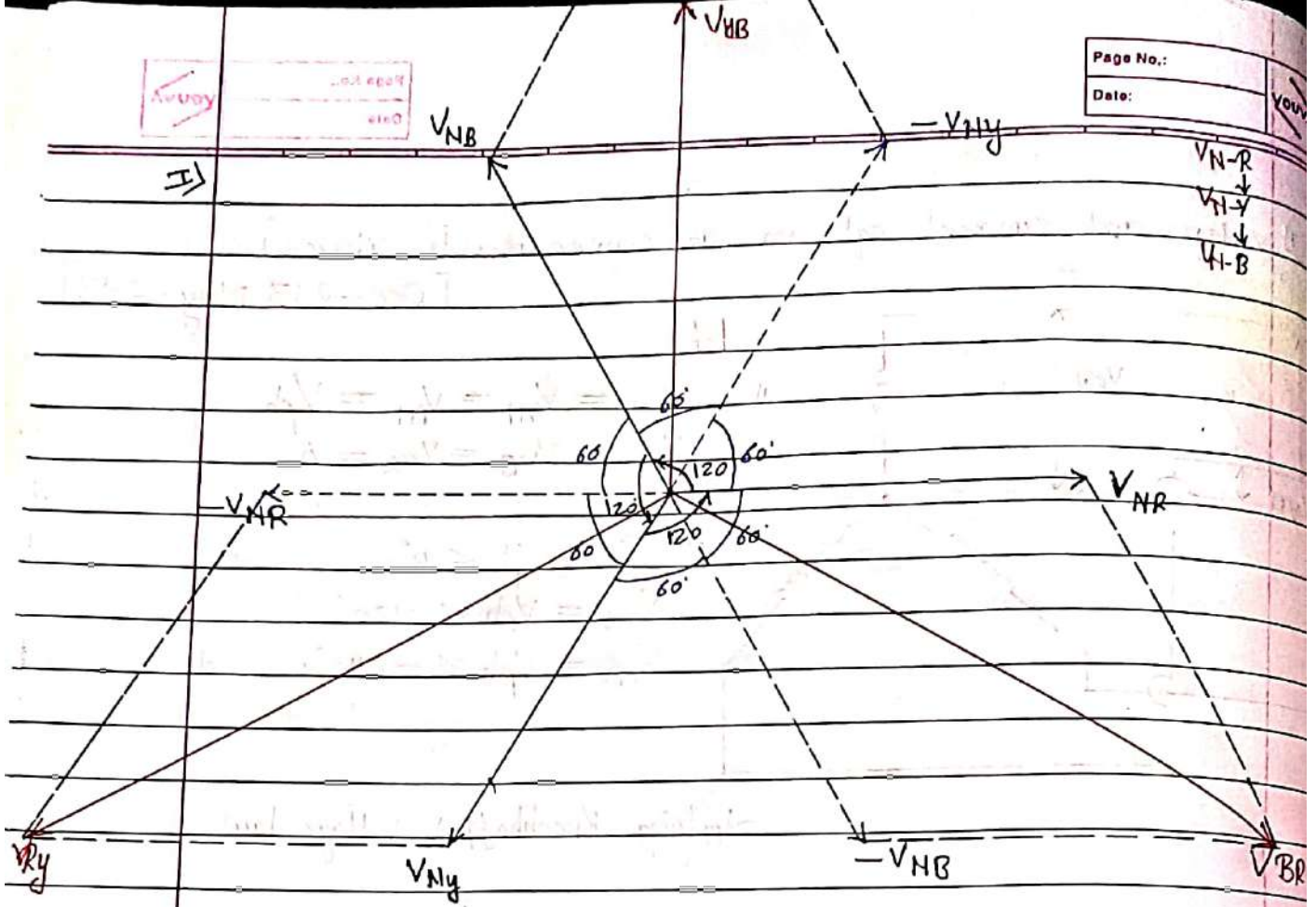
(1) 30.

$$P = V_L \cdot I_L \cdot \cos \phi$$

$$P = \sqrt{3} \cdot V_{ph} \cdot I_{ph} \cos \phi \quad W$$

$$Q = \sqrt{3} \cdot V_{ph} \cdot I_{ph} \sin \phi \quad VAR$$

$$S = \sqrt{3} \cdot V_{ph} \cdot I_{ph} \quad VA$$



3) Impedance Relⁿ * Δ Δ^L

1) Star connection

2) Delta connection

V_L = line voltage

V_L = line voltage

I_L = line current

I_L = line current

Z_Y = Impedance / Phase

Z_Δ = Impedance / Phase

$$\rightarrow V_{ph} = \frac{1}{\sqrt{3}} V_L$$

$$\rightarrow V_{ph} = \frac{V_L}{\sqrt{3}}$$

$$\rightarrow I_{ph} = I_L$$

$$\rightarrow I_{ph} = \frac{1}{\sqrt{3}} I_L$$

$$\rightarrow Z_{ph} = \frac{V_{ph}}{I_{ph}}$$

$$\rightarrow Z_{ph} = \frac{V_{ph}}{I_{ph}}$$

$$\rightarrow Z_Y = Z_Y = \frac{V_{ph}}{I_{ph}}$$

$$\rightarrow Z_\Delta = \frac{V_{ph}}{I_{ph}}$$

$$\therefore Z_Y = \frac{1}{\sqrt{3}} \cdot \frac{V_L}{I_L}$$

$$\therefore Z_\Delta = \frac{V_L}{\frac{1}{\sqrt{3}} I_L}$$

$$\therefore Z_Y = \frac{1}{\sqrt{3}} \cdot \frac{V_L}{I_L}$$

$$\therefore Z_\Delta = \sqrt{3} \cdot \frac{V_L}{I_L}$$

$$\therefore \sqrt{3} \cdot Z_Y = \frac{V_L}{I_L} \quad - (1)$$

$$\therefore Z_\Delta = \sqrt{3} \times \sqrt{3} \cdot Z_Y$$

$$Z_\Delta = 3 Z_Y$$

$$\therefore Z_Y = \frac{1}{3} Z_\Delta$$

4) Relation betⁿ Power in Delta and Star Systems.

i) $P = \sqrt{3} \cdot V_L \cdot I_L \cdot \cos\phi$ 2) Δ^k

*
 $\rightarrow V_{ph} = \frac{V_L}{\sqrt{3}} \rightarrow V_L = V_{ph}$

$\rightarrow I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{1}{\sqrt{3}} \cdot \frac{V_L}{Z_{ph}} \rightarrow I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{V_L}{Z_{ph}}$

$\rightarrow P = \sqrt{3} \cdot V_L \cdot I_L \cdot \cos\phi \rightarrow I_L = \sqrt{3} \cdot I_{ph} = \frac{\sqrt{3} \cdot V_L}{Z_{ph}}$

$P = \sqrt{3} \times V_L \times \frac{1}{\sqrt{3}} \cdot \frac{V_L}{Z_{ph}} \times \cos\phi \rightarrow P = \sqrt{3} \cdot V_L \cdot I_L \cdot \cos\phi$

$P_* = \frac{V_L^2}{Z_{ph}} \cdot \cos\phi$ - (1)

$P = \sqrt{3} \times V_L \times \frac{\sqrt{3} \cdot V_L}{Z_{ph}} \times \cos\phi$

$P_A = 3 \frac{V_L^2}{Z_{ph}} \cdot \cos\phi$ - (2)

$P_A = 3 \cdot P_*$ [From (1)]

$P_* = \frac{1}{3} P_A$

5) Measurement of Power by Two-wattmeter Method

i) A balanced star-connected load. The load may be assumed to be inductive.

→ Let V_{RN} , V_{YN} and V_{BN} be the three-phase voltages.

→ I_R , I_Y and I_B be the phase currents.

→ The phase current will lag behind their resp. phase voltages by angle ϕ .

→ Current through current coil of $W_1 = I_R$

→ Voltage across voltage coil of

$$\textcircled{1} \quad W_1 = V_{RB} = V_{RN} + V_{NB}$$

$$W_1 = V_{RB} = V_{RN} - V_{BN}$$

$$\Rightarrow W_1 = V_{RB} \cdot I_R \cdot \cos[30 - \phi]$$

$$\downarrow \quad \downarrow$$

$$V_h \quad I_h$$

$$\Rightarrow \boxed{W_1 = V_h \cdot I_h \cdot \cos[30 - \phi]} \quad \text{--- (1)}$$

$$\textcircled{2} \quad W_2 = I_Y$$

$$W_2 = V_{YB} = V_{YN} + V_{NB}$$

$$W_2 = V_{YB} = V_{YN} - V_{BN}$$

$$\Rightarrow W_2 = V_{YB} \cdot I_Y \cdot \cos[30 + \phi]$$

$$\downarrow \quad \downarrow$$

$$V_h \quad I_h$$

$$\Rightarrow \boxed{W_2 = V_h \cdot I_h \cdot \cos[30 + \phi]} \quad \text{--- (2)}$$

$$\text{but } I_R = I_Y = I_L$$

$$V_{RB} = V_{YB} = V_L$$

$$W_T = W_1 + W_2$$

$$= V_L \cdot I_L [\cos(30^\circ - \phi) + \cos(30^\circ + \phi)]$$

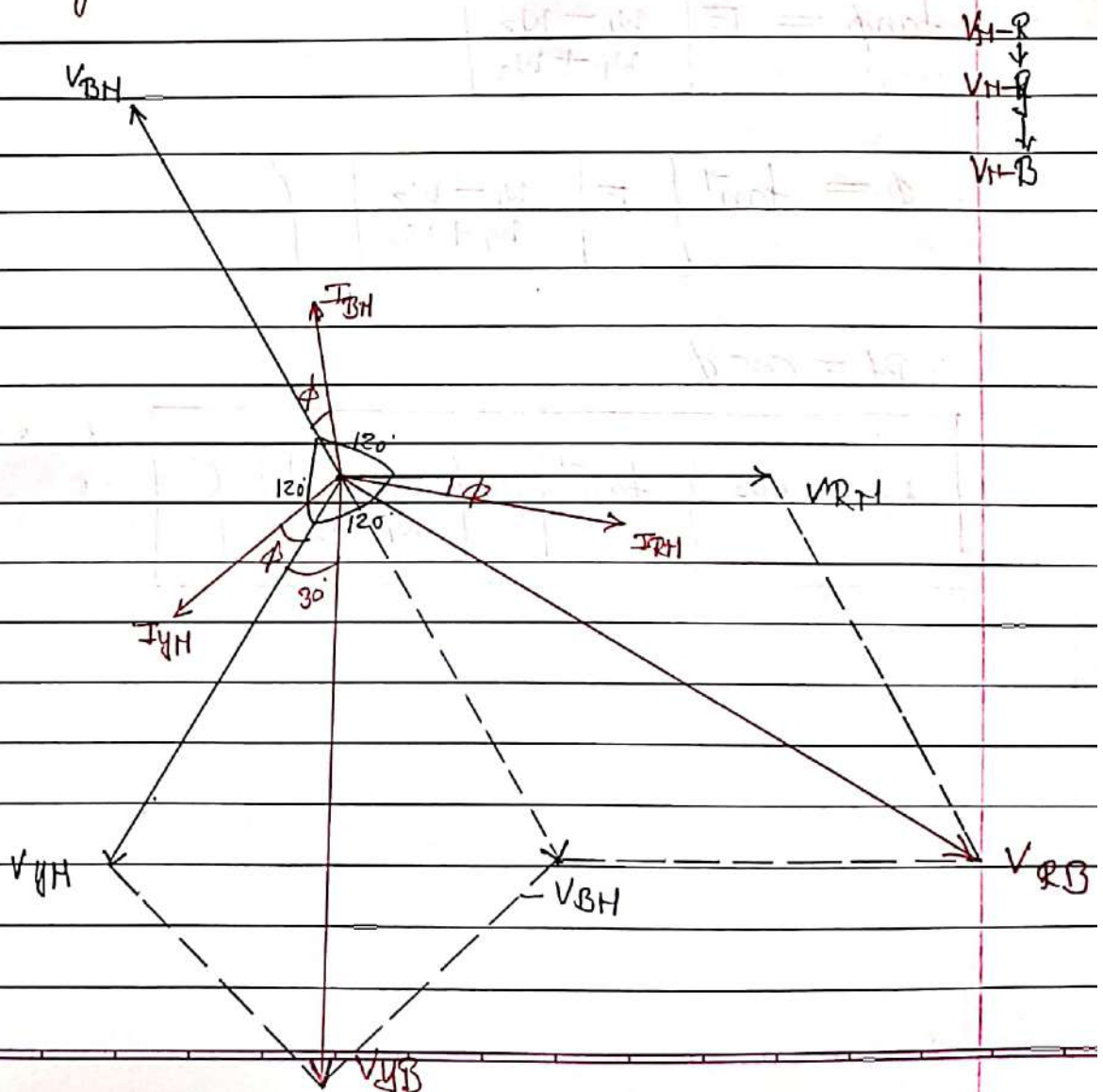
$$= V_L \cdot I_L [2 \cos 30^\circ \cdot \cos \phi]$$

$$= V_L \cdot I_L \left[2 \times \frac{\sqrt{3}}{2} \cdot \cos \phi \right]$$

$$= V_L \cdot I_L [\sqrt{3} \cdot \cos \phi]$$

$$W_T = \sqrt{3} \cdot V_L \cdot I_L \cdot \cos \phi$$

Phasor diagram



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1) Measurement of Power factor by Two-wattmeter Method

i) lagging power factor

$$W_1 = V_L \cdot I_L \cdot \cos [30 - \phi]$$

$$W_2 = V_L \cdot I_L \cdot \cos [30 + \phi]$$

$$\therefore W_1 > W_2$$

$$\rightarrow W_1 + W_2 = \sqrt{3} \cdot V_L \cdot I_L \cdot \cos \phi$$

$$\rightarrow W_1 - W_2 = V_L \cdot I_L \cdot \sin \phi$$

$$\rightarrow \frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L \cdot I_L \cdot \sin \phi}{\sqrt{3} \cdot V_L \cdot I_L \cdot \cos \phi}$$

$$\rightarrow \tan \phi = \sqrt{3} \cdot \frac{W_1 - W_2}{W_1 + W_2}$$

$$\therefore \phi = \tan^{-1} \left\{ \sqrt{3} \cdot \frac{W_1 - W_2}{W_1 + W_2} \right\}$$

$$\therefore \text{pf} = \cos \phi$$

$$\boxed{\text{pf} = \cos \left\{ \tan^{-1} \left[\sqrt{3} \cdot \frac{W_1 - W_2}{W_1 + W_2} \right] \right\}}$$

7) Max leading power factor

$$i) w_1 = V_L \cdot I_L \cdot \cos [30 + \phi]$$

$$w_2 = V_L \cdot I_L \cdot \cos [30 - \phi]$$

$$\therefore w_1 < w_2$$

$$\rightarrow w_1 + w_2 = \sqrt{3} \cdot V_L \cdot I_L \cdot \cos \phi$$

$$\rightarrow w_1 - w_2 = -V_L \cdot I_L \cdot \sin \phi$$

$$\rightarrow \frac{w_1 - w_2}{w_1 + w_2} = \frac{-V_L \cdot I_L \cdot \sin \phi}{\sqrt{3} \cdot V_L \cdot I_L \cdot \cos \phi}$$

$$\rightarrow \tan \phi = -\sqrt{3} \left[\frac{w_1 - w_2}{w_1 + w_2} \right]$$

$$\rightarrow \therefore \phi = \tan^{-1} \left\{ -\sqrt{3} \left[\frac{w_1 - w_2}{w_1 + w_2} \right] \right\}$$

$$\therefore \text{Pf} = \cos \phi$$

$$\therefore \text{Pf} = \cos \left\{ \tan^{-1} \left[-\sqrt{3} \left(\frac{w_1 - w_2}{w_1 + w_2} \right) \right] \right\}$$

4) what are the Advantages of a three-phase system over a single-phase system?

i) In a single-phase system, The instantaneous power is fluctuating in nature.

2) In a three-phase system, it is constant at all times.

3) The output of a three-phase system is greater than that of a single-phase system.

4) Transmission and distribution of a three-phase system is cheaper than that of a single-phase system.

5) Three-phase motors are more efficient and have higher factors than single-phase motors of the same frequency.

6) Three-phase motors are self-starting whereas single-phase motors are not self-starting.

5) Some Definitions.

i) Phase Sequence

→ The sequence in which the voltages in the three phases reach the maximum positive value is called the phase sequence or phase order. the phase sequence is R-Y-B.

2) Balanced Load

→ The load is said to be balanced if load connects across the three phases are identical, i.e. all the loads have the same magnitude and power factor.

3] Symmetrical or Balanced system

→ A three-phase system is said to be balanced if the

a) Voltages in the three-phases are equal in magnitude and differ in phase from one another by 120° .

and
b) Currents in the three-phases are equal in magnitude and differ in phase from one another by 120° .

4] Line Quantities

a) Line voltage.

→ The voltage available between any pair of terminals or lines is called the line voltage.

b) Line current.

→ The current flowing through each line is called the line current.

5] Phase Quantities

a) Phase voltage.

→ The voltage induced in each winding is called the phase voltage.

b) Phase current.

→ The current flowing through each winding is called the phase current.

Q] Difference between Delta and Star connections.

Delta Connection

Star connection

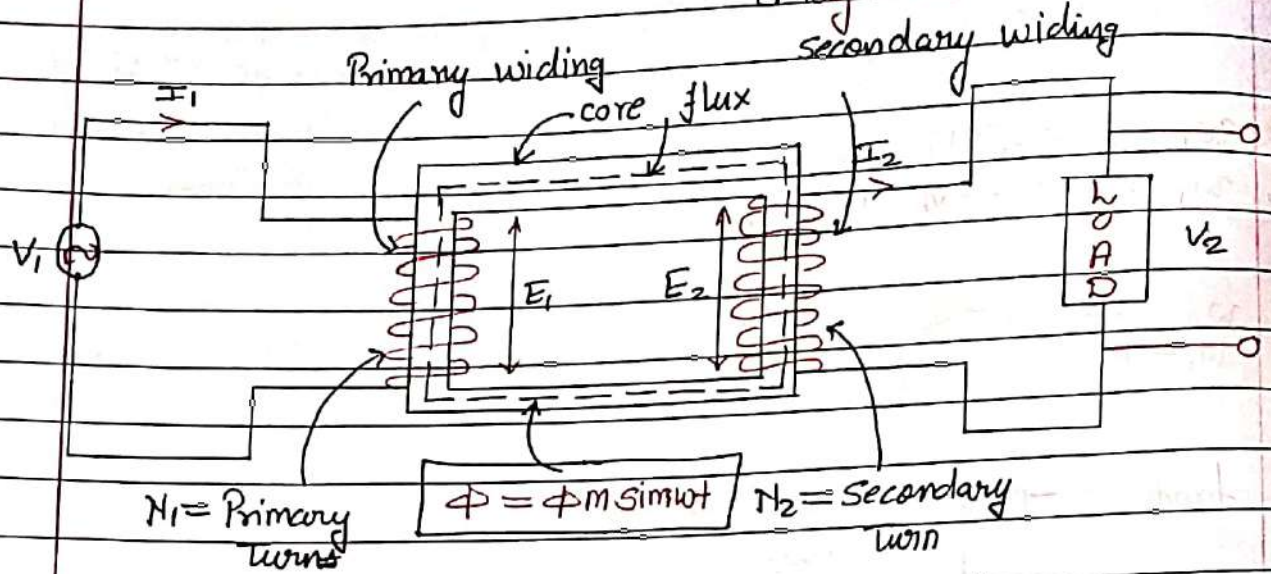
- | | |
|--|--|
| 1] $V_L = V_{ph}$ | 1] $V_L = \sqrt{3} V_{ph}$ |
| 2] $I_L = \sqrt{3} I_{ph}$ | 2] $I_L = I_{ph}$ |
| 3] line current lags behind the respective phase current by 30° . | 3] line voltage leads the respective phase voltage by 30° . |
| 4] Power in delta connection is 3 times of the power in star connection. | 4] Power in star connection is one-third of power in delta connection. |
| 5] The phasor sum of all the phase voltages is zero. | 5] The phasor sum of all the phase currents is zero. |

chap-6 Single-phase Transformers

[4M] + [6M] *** Explain the working principle of a transformer.
emcl] Derive emf equation for single-phase transformer.

[Dec-12, 14, 18, 17]

[May-13, 15]



$$v = V_m \sin \omega t$$

$$\phi = \phi_m \sin \omega t$$

$$e_1 = -N_1 \frac{d\phi}{dt} \quad [\because \omega = 2\pi f]$$

$$e_{1\max} = 2\pi f \phi_m N_1$$

$$e = -N \frac{d\phi}{dt}$$

$$E_{1\max} = \frac{e_{1\max}}{\sqrt{2}} = \frac{2\pi f \phi_m N_1}{\sqrt{2}}$$

$e = \text{EMF}$

$N = \text{No. of turns}$

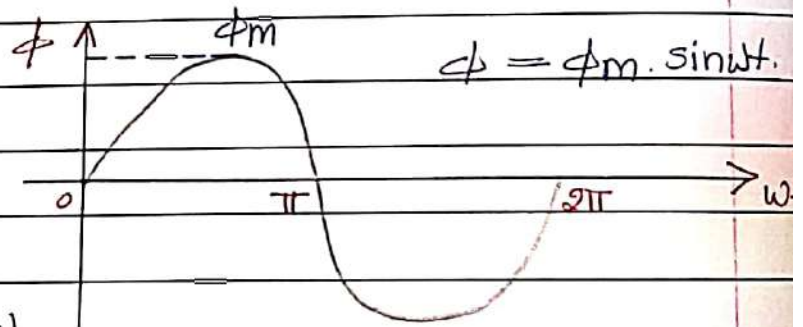
$\frac{d\phi}{dt} = \text{Rate of change of } \phi \text{ w.r. to 't'}$

$$E_1 = 4.44 f \phi_m N_1 \rightarrow \text{P. winding}$$

$$E_2 = 4.44 f \phi_m N_2 \rightarrow \text{S. winding}$$

$$e_1 = -N_1 \frac{d\phi}{dt}$$

$$e_1 = -N_1 \frac{d[\phi_m \sin \omega t]}{dt}$$



$$= -N_1 \phi_m \cos \omega t \cdot (\omega)$$

$$= -N_1 \phi_m \omega \sin [90^\circ - \omega t]$$

$$= + N_1 \phi_m \omega \sin [\omega t - 90^\circ] \rightarrow [\text{Max. o/p}]$$

$$e_1 = N_1 \phi_m \omega$$

2) Transformation Ratio [K]

$$\rightarrow E_1 = 4.44 f \phi_m N_1$$

$$E_2 = 4.44 f \phi_m N_2$$

$$K = \frac{E_2}{E_1}$$

$$K = \frac{4.44 f \phi_m N_2}{4.44 f \phi_m N_1}$$

$$K = \frac{N_2}{N_1}$$

$$K = \frac{E_2}{E_1} = \frac{N_2}{N_1} \quad - (1)$$

$\rightarrow$ Let consider loss in E and V to be negligible.

$$E_1 \cong V_1 \quad \text{and}$$

$$E_2 \cong V_2$$

$$\therefore K = \frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1} \quad - (2)$$

* let consider I/P and o/p to be same.

$$V_1 I_1 = V_2 I_2$$

$$\therefore \frac{V_2}{V_1} = \frac{I_1}{I_2}$$

1) For step-down transformer

$$\frac{N_2}{N_1} < 1$$

$$\Rightarrow N_2 < N_1 \quad \text{i.e. } K < 1$$

2) For step-up transformer

$$\therefore K = \frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} \quad \left[\frac{N_2}{N_1} > 1 \right] \Rightarrow N_2 > N_1 \quad \text{i.e. } K > 1$$

K $\rightarrow$ Transformation Ratio.

3) For isolation transformer

$$\frac{N_2}{N_1} = 1$$

$$\Rightarrow N_2 = N_1 \quad \text{i.e. } K = 1$$

1) Definition of a Transformer

→ A Transformer is a static device which can transfer electrical energy from one circuit to another circuit without change of frequency.

2) Turn ratio

→ The ratio of Primary to secondary turns is called turn ratio.

$$\frac{N_1}{N_2}$$

2) Transformation Ratio K

→ The ratio of Secondary to Primary turns is called transformation ratio.

$$K = \frac{N_2}{N_1}$$

Q1) what are the losses in transformer? Explain why the ratings of transformer in KVA not in KW?
[04-May-2017, 04-May-2019]

OR

Q2) Explain the various losses of a single-phase transformer.
[04-May-2017]

OR

Q3) With the help of equivalent circuit of a single-phase transformer show how total copper loss can be represented in primary of a transformer.
[04-May-2018]

Ans.

→ There are two types of losses in a transformer.

1) Iron loss

2) Copper loss

1) Iron loss.

→ This loss is due to the reversal of flux in the core. The flux set-up in the core is nearly constant.

2) Copper loss.

→ This loss is due to the resistance of Primary and secondary windings.

$$W_{cu} = I_1^2 R_1 + I_2^2 R_2$$

∴ R_1 = Primary winding resistance

R_2 = secondary winding resistance

→ Copper loss depends upon the load on the transformer and is proportional to square of load current at KVA rating of the transformer.

[OUM-2014]

3] Transformation Rating

i) what are the losses in a transformer? Explain why the rating of transformer is expressed in KVA not in KW.

→ Rating of a transformer indicates the output from it. But for a transformer, load is not fixed and its power factor goes on changing. Hence, Rating is not expressed in terms of power but in terms of product of voltage and current, called VA Rating. This rating is generally expressed in KVA.

$$\text{KVA rating of a transformer} = \frac{V_1 I_1}{1000} = \frac{V_2 I_2}{1000}$$

$$\text{i.e. Full-load primary current} = I_1 = \frac{\text{KVA rating} \times 1000}{V_1}$$

$$\text{Full-load secondary current} = I_2 = \frac{\text{KVA rating} \times 1000}{V_2}$$

b) what are Assumptions for an ideal transformer?

04-11-2013

- Ans. 1) Winding resistance are negligible.
 2) The copper losses are negligible.
 3) The core or iron losses are negligible.
 4) All the flux setup by primary links the secondary windings i.e. no magnetic leakage flux.
 5) The magnetization curve for the core is linear.

04-M-Dec-2017

#> Differentiate between shell-type and core-type transformers.

Core-Type Transformer

- 1) It has a single magnetic circuit.
- 2) It is easy to repair.
- 3) The winding encircles the core.
- 4) It consists of cylindrical windings.
- 5) It is Preferred for low-voltage transformers.

Shell-Type Transformer

- 1) It has two magnetic circuit.
- 2) It is not easy to repair.
- 3) The core encircles most part of the winding.
- 4) It consists of sandwich-type windings.
- 5) It is Preferred for high-voltage transformers.

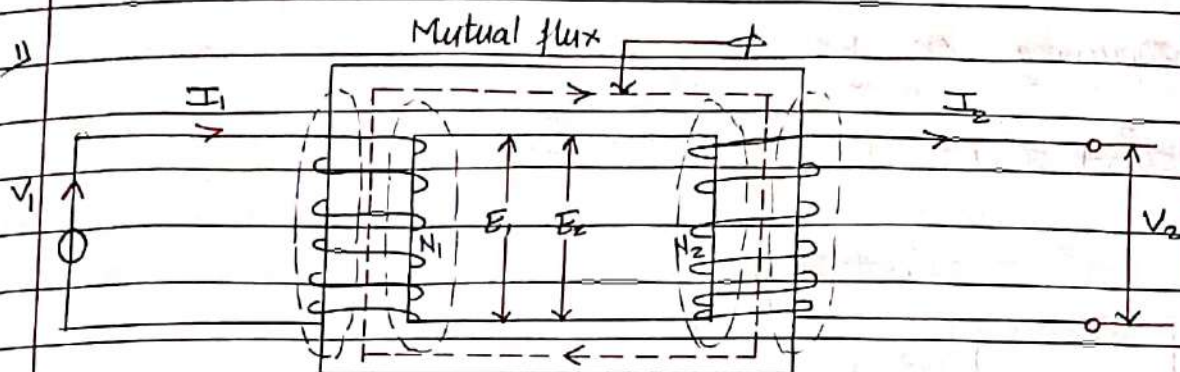
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04-May-2016, 08-May-2017, Dec-2013, 2015, May-2015

4) Draw and Explain the phasor diagram of a single-phase transformer on No-load.



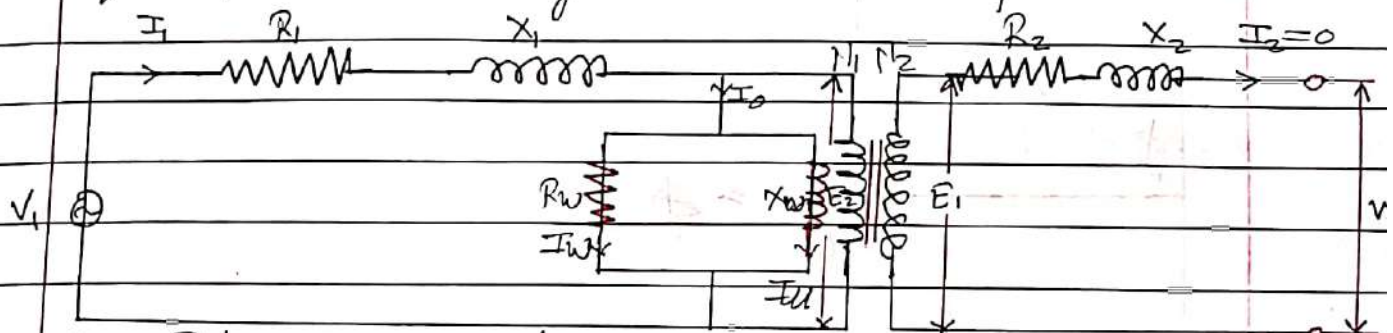
$I_1 =$ No load Primary current (I_0)

I_0 has two components.

1) Magnetising component of I_0 (I_u)

2) Power component of I_0 (I_w)

2) Equivalent circuit diagram of a transformer.



$I_1 =$ Primary current.

$V_1 =$ Primary voltage.

$R_1 =$ Primary winding resistance.

$X_1 =$ Primary leakage Reactance.

$I_0 =$ No Load Primary current.

$I_w =$ core loss component of I_0 .

$I_u =$ Magnetising component of I_0 .

$R_w =$ core loss Resistance

$X_m =$ Magnetising Reactance

$E_1 =$ Primary induced E.M.

$E_2 =$ Secondary induced E.M.

$N_1 =$ No. of Primary wdg. turns

$N_2 =$ No. of secondary wdg. turns

$R_2 =$ secondary winding resistance

$X_2 =$ secondary leakage Reactance

$V_2 =$ Secondary voltage

$$I_W = I_o \cos \phi_o$$

$$I_M = I_o \sin \phi_o$$

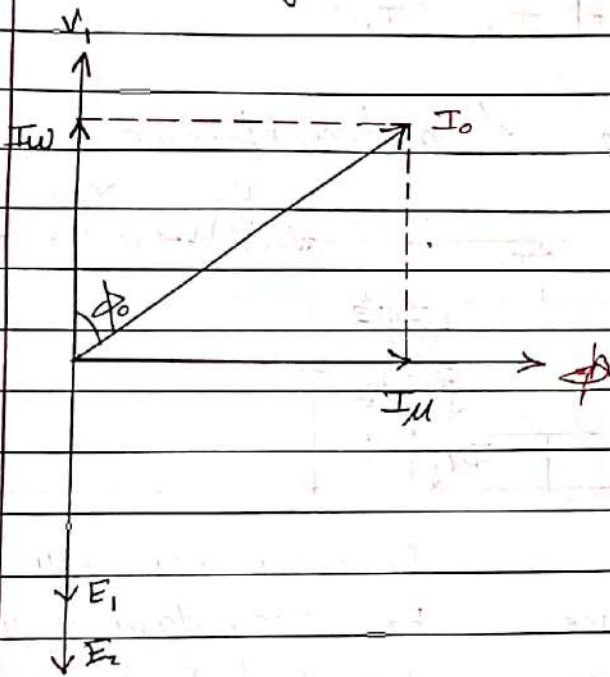
Squaring on b.s.

$$I_W^2 + I_M^2 = I_o^2 [\sin^2 \phi_o + \cos^2 \phi_o]$$

$$\therefore I_o^2 = I_W^2 + I_M^2$$

$$\therefore I_o = \sqrt{I_W^2 + I_M^2}$$

2) Phasor Diagram

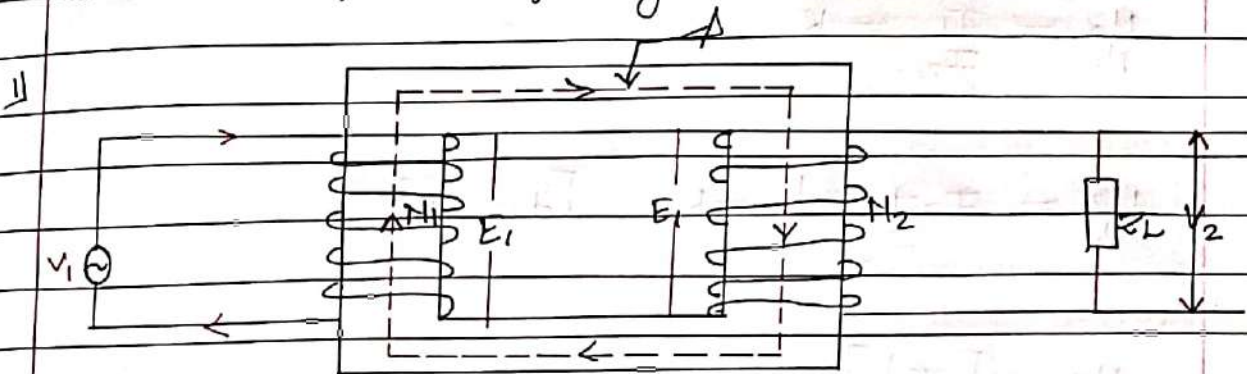


[04-M-Dec-18, 08-M-Dec-18, 06-May-2019]

1) Draw and Explain the phasor diagram for practical transformer connected to lagging power factor load. OR [06-M-Dec-2015]

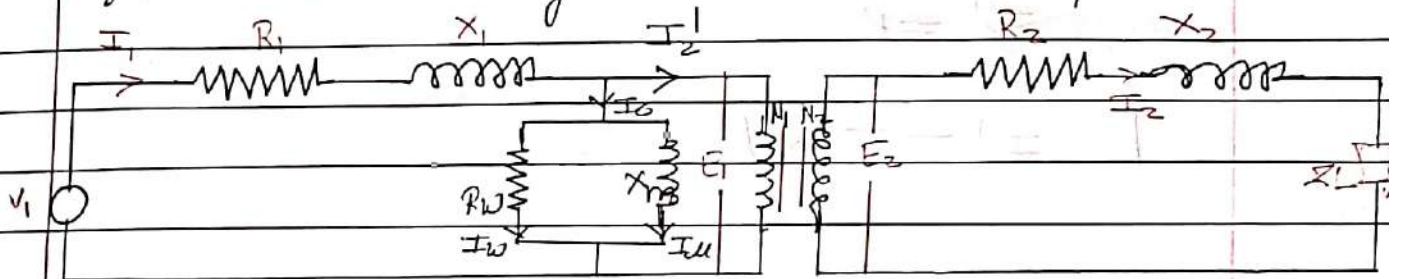
2) Develop the Phasor diagram of a single phase transformer supplying to a resistive load. OR [08-M-Dec-2016]

3) Draw the Phasor diagram of a single phase non ideal transformer feeding a resistive load. [04-Dec-19]



$$\frac{N_2}{N_1} = \frac{E_1}{E_2} = K$$

2) Equivalent circuit diagram of 1φ-transformer on load



I_1 = Primary current

N_1 = Primary turns

V_1 = Primary voltage

N_2 = Secondary turns

R_1 = Primary Resistance

E_1 = Primary induced EMF

X_1 = Primary Reactance

E_2 = Secondary induced EMF

I_0 = No Load Primary current

R_2 = Secondary Resistance

I_W = core loss comp. of I_0

X_2 = secondary leakage Reactance

I_M = Magnetising comp. of I_0

I_2 = secondary current

R_W = core loss Resistance

V_2 = secondary voltage

X_m = magnetising Reactance

I_1' = comp. of I_1 to nullify effect of ϕ_2

$$V_1 = I_1 R_1 + I_1 X_1 + (-E_1)$$

$$\begin{cases} V_1 = -E_1 \\ E_1 = E_2 \end{cases}$$

$$V_2 = I_2 R_2 + I_2 X_2 + E_2$$

$$E_2 = I_2 R_2 + I_2 X_2 + V_2 \quad \{V_2 = E_2\}$$

$$\therefore I_1 = I_0 + I_2' \quad \text{--- (1)}$$

$$\frac{N_2}{N_1} = \frac{I_1}{I_2} = K$$

$$\frac{N_2}{N_1} = \frac{I_0 + I_2'}{I_2} = K \quad [\text{From (1)}]$$

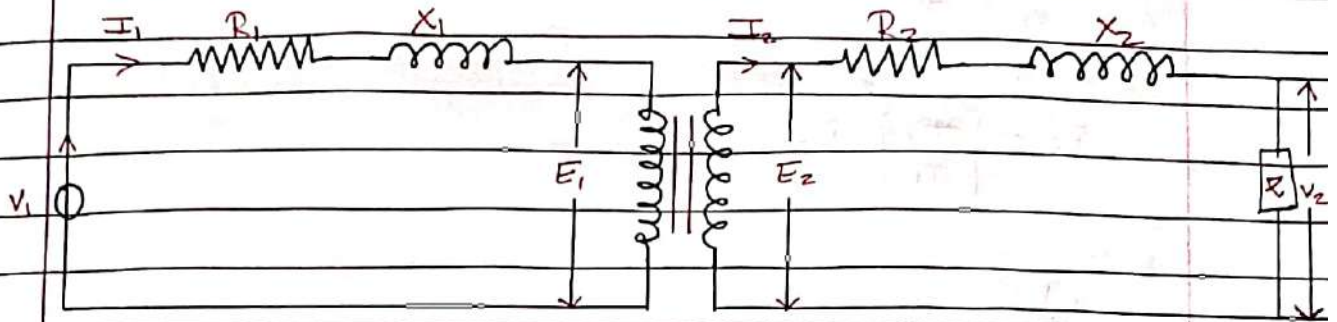
$$\therefore \frac{N_2}{N_1} = \left[\frac{I_2'}{I_2} \right] = K$$

$$\therefore \frac{I_2'}{I_2} = K$$

$$\boxed{\therefore I_2' = I_2 \cdot K}$$

[05-MI-Dec-2016]

6) Derive the Equivalent circuit of a 1-Phase transformer.



I_1 = Primary current.

I_2 = Secondary current.

R_1 = Primary Resistance.

R_2 = secondary Resistance.

X_1 = Primary Reactance.

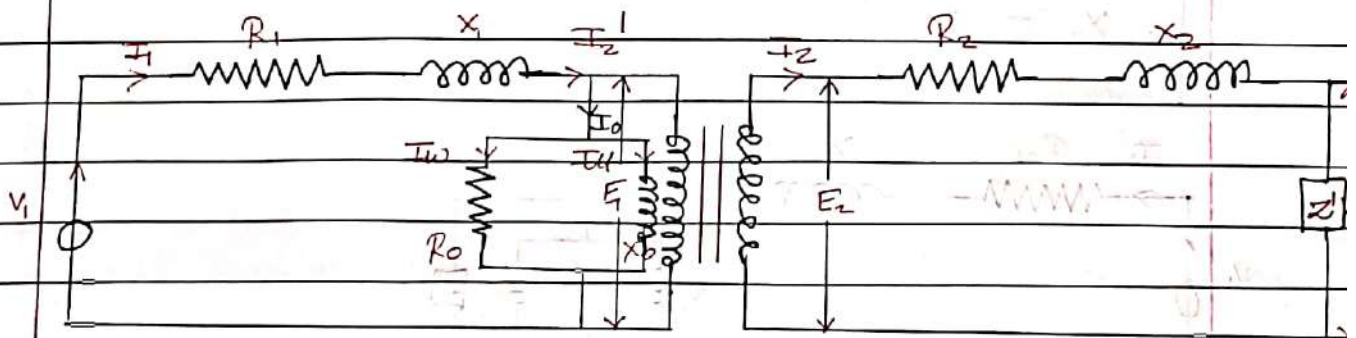
X_2 = Secondary Reactance.

V_1 = Primary Voltage.

V_2 = secondary Voltage.

E_1 = Primary induced EMF

E_2 = secondary induced EMF.



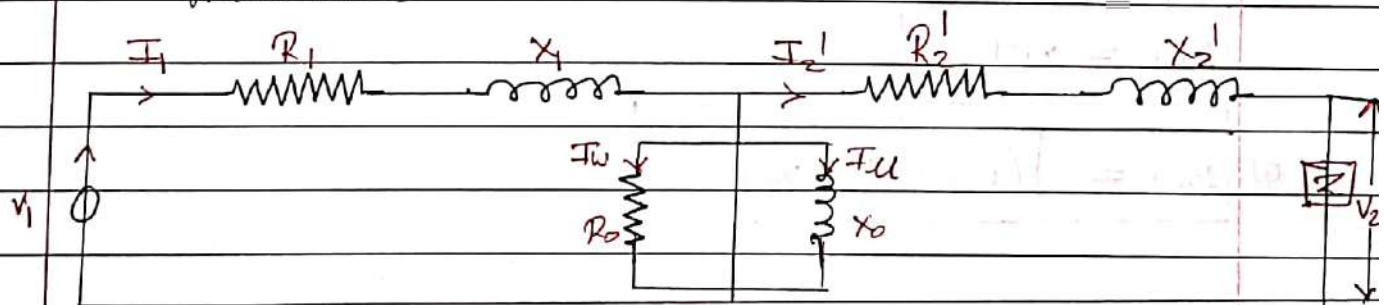
I_0 = No load Primary current.

I_w = core loss component of I_0

I_m = Magnetising comp. of I_0 .

R_0 = Resistance

X_0 = Inductance.



$$I_1^2 R_2' = I_2^2 \cdot R_2$$

$$\therefore R_2' = \frac{I_2^2}{I_1^2} \cdot R_2$$

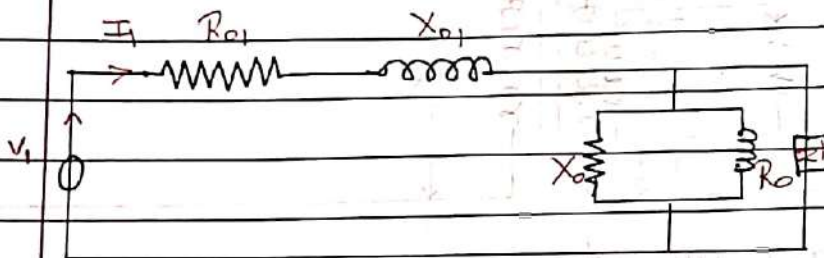
$$\therefore R_2' = \left[\frac{I_2}{I_1} \right]^2 \cdot R_2$$

$$\therefore R_2' = R_2 \left[\frac{I_1}{I_2} \right]^2$$

$$\therefore R_2' = \frac{R_2}{K^2} \quad \left\{ \because K = \frac{I_1}{I_2} \right\}$$

Similarly,

$$\therefore X_2' = \frac{X_2}{K^2}$$



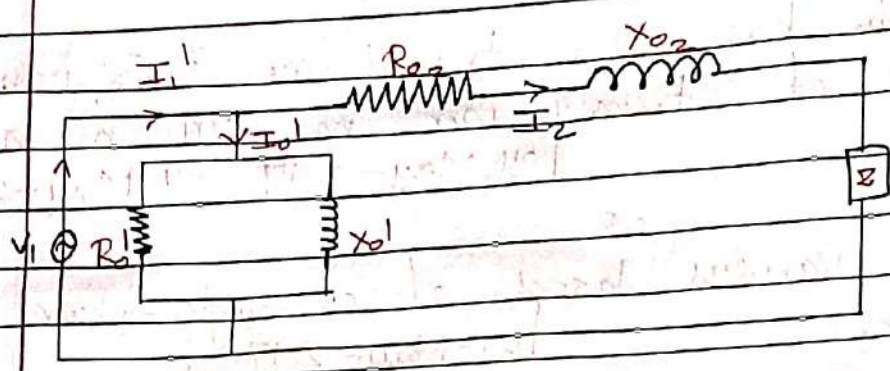
$$1) R_{01} = R_1 + R_2'$$

$$R_{01} = R_1 + \frac{R_2}{K^2}$$

$$2) X_{01} = X_1 + X_2'$$

$$X_{01} = X_1 + \frac{X_2}{K^2}$$

$$3) Z_{01} = \sqrt{(R_{01})^2 + (X_{01})^2}$$



$$R_{02} = R_1' + R_2$$

$$\therefore R_1' = k^2 R_1$$

$$R_{02} = k^2 R_1 + R_2$$

$$R_{02} = k^2 \left[R_1 + \frac{R_2}{k^2} \right]$$

$$R_{02} = k^2 \cdot R_{01}$$

Similarly,

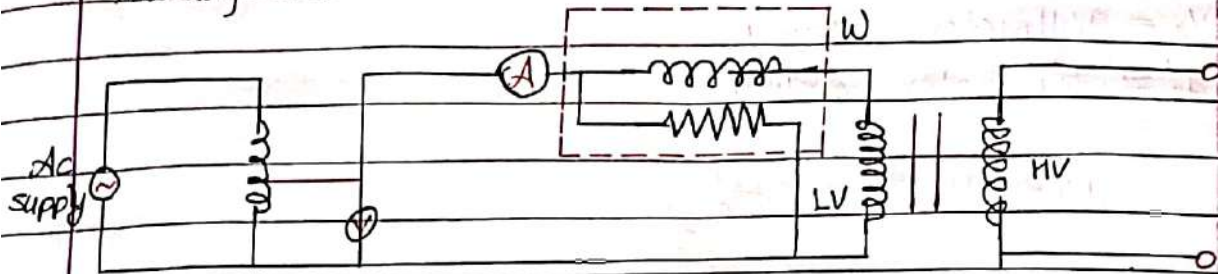
$$X_{02} = k^2 \cdot X_{01}$$

$$\rightarrow Z_{02} = \sqrt{(R_{02})^2 + (X_{02})^2}$$

$$\therefore Z_{02} = \sqrt{(k^2 R_{01})^2 + (k^2 X_{01})^2}$$

$$\therefore Z_{02} = k^2 \cdot Z_{01}$$

Q1 with the help of a neat diagram explain how open circuit test is conducted on a single-phase transformer. [6M-DEC-2014]



→ OC Test is performed on low-voltage winding keeping high voltage winding open circuited.

→ W_i = No load losses / core losses.

V_1 = No load Primary voltage

I_0 = No load Primary current.

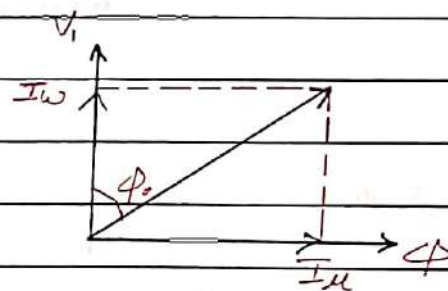
→ Copper losses are negligible and the wattmeter indicates iron loss.

relations: → when meters are connected on the Primary side

W_i = wattmeter reading

V_1 = voltage reading

I_0 = Ammeter reading



$$W_i = V_1 \cdot I_0 \cdot \cos \phi_0$$

$$\cos \phi_0 = \frac{W_i}{V_1 I_0}$$

$$R_w = R_0 = \frac{V_1}{I_w}$$

$$I_w = I_0 \cos \phi_0$$

$$I_u = I_0 \sin \phi_0$$

$$X_m = X_0 = \frac{V_1}{I_u}$$

2) When meters are connected on the secondary side

W_i = Wattmeter reading

V_2 = Voltmeter reading

I_o' = Ammeter reading

$$W_i = V_2 I_o' \cos \phi_o'$$

$$\cos \phi_o' = \frac{W_i}{V_2 I_o'}$$

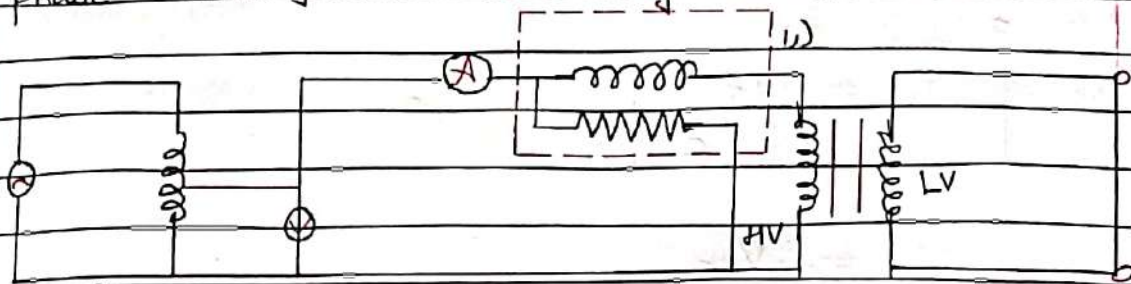
$$I_w' = I_o' \cos \phi_o'$$

$$I_u' = I_o' \sin \phi_o'$$

$$R_w = R_o = \frac{V_2}{I_w'}$$

$$X_m = X_o = \frac{V_2}{I_u'}$$

Q1 with the help of a neat diagram explain how short circuit test is conducted on a single phase transformer. [06-May-2015]



→ Normally SC Test is Performed on High Voltage winding short circuiting low-voltage winding.

→ W_{sc} = short circuit Power.

V_{sc} = short circuit voltage.

I_{sc} = short circuit current.

→ wattmeter will read copper losses because of high current passing through windings.

relations $\Rightarrow$ if W_{sc} = wattmeter reading

V_{sc} = voltmeter reading

I_{sc} = ammeter reading.

if when meters are connected on the Primary side

$$W_{sc} = I_{sc}^2 R_{01}$$

$$R_{01} = \frac{W_{sc}}{I_{sc}^2}$$

$$Z_{01} = \frac{V_{sc}}{I_{sc}}$$

$$X_{01} = \sqrt{(Z_{01})^2 - (R_{01})^2}$$

2) When meters are connected on the secondary side

$$W_{SC} = I_{SC}^2 R_{02}$$

$$\therefore R_{02} = \frac{W_{SC}}{I_{SC}^2}$$

$$Z_{02} = \frac{V_{SC}}{I_{SC}}$$

$$\therefore X_{02} = \sqrt{(Z_{02})^2 - (R_{02})^2}$$

III) Define voltage regulation and Derive its expression.

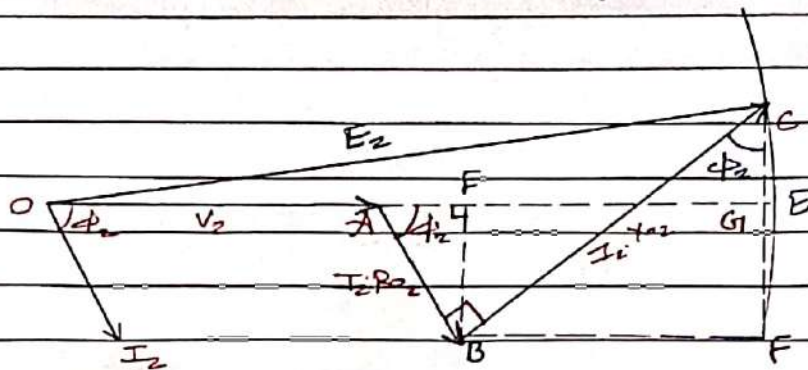
Ans. → When a transformer is loaded, with a constant supply voltage, the change in terminal voltage from No-load to full load with respect to no load voltage is called voltage regulation of transformer.

$$\text{Regulation} = \frac{\text{Secondary terminal voltage on No-load} - \text{Secondary terminal voltage on full-load condition}}{\text{Secondary terminal voltage on No-load}}$$

$$\text{Regulation} = \frac{E_2 - V_2}{E_2}$$

$$\% \text{ Regulation} = \frac{E_2 - V_2}{E_2} \times 100$$

2) Expression for voltage Regulation



$$\begin{aligned} \rightarrow E_2 - V_2 &= AE \approx AG \\ &= AF + EG \\ &= AF + BF \end{aligned}$$

$$1) \cos \phi_2 = \frac{AF}{I_2 \cdot R_{02}}$$

$$2) \sin \phi_2 = \frac{BF}{I_2 \cdot X_{02}}$$

$$\therefore AF = I_2 \cdot R_{02} \cdot \cos \phi_2$$

$$\therefore BF = I_2 \cdot X_{02} \cdot \sin \phi_2$$

$$3) E_2 - V_2 = AF + BF$$

$$E_2 - V_2 = I_2 R_{02} \cdot \cos \phi_2 + I_2 \cdot X_{02} \cdot \sin \phi_2 \quad \left[\text{lagging P.f.} \right]$$

$$E_2 - V_2 = I_2 R_{02} \cdot \cos \phi_2 - I_2 \cdot X_{02} \cdot \sin \phi_2 \quad \left[\text{Leading P.f.} \right]$$

$$\% \text{ V.R.} = \frac{E_2 - V_2}{E_2} \times 100$$

$$\% \text{ V.R.} = \frac{I_2 R_{02} \cos \phi_2 \pm I_2 \cdot X_{02} \sin \phi_2}{E_2} \times 100$$

Electrical Machines

Notes by:-
Prof. Ashish Vanmali

* Introduction :-

From construction point of view, a d.c. generator is identical with a d.c. motor. However, there is a difference in their operations.

- When mechanical energy is given as an input to the machine, and if it delivers electrical energy as its output, the machine becomes a generator.
- When electrical energy is given as an input to the machine, and if it delivers mechanical energy as its output, the machine becomes a motor.

* Principle of working :-

1) Generator :-

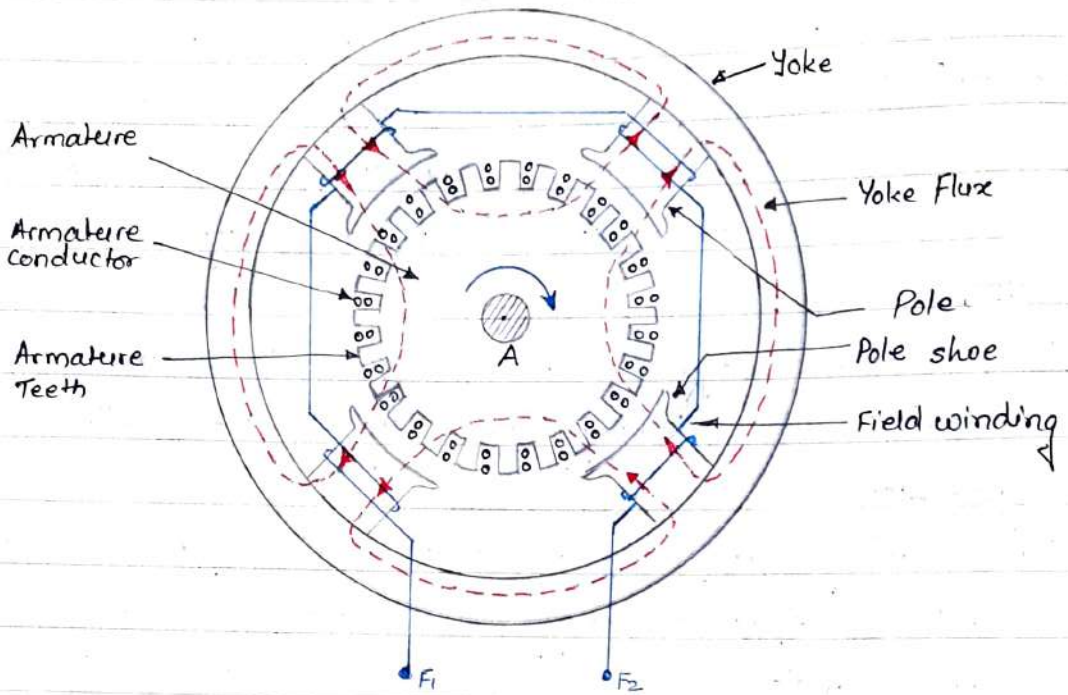
A generator works on the principle of dynamically induced emf. According to this principle, whenever a conductor is moved in a magnetic field in a direction at right angle to the field, it cuts flux line and a emf is induced in it.

2) Motor :-

A dc motor works on the principle that a current carrying conductor placed in magnetic field experiences a mechanical force whose direction is given by Fleming's Left hand rule and whose magnitude is $F = B * I * l$ where B is the flux density, I is the current carried by the conductor and l is the length of the conductor.

* Construction of DC Machines :-

A dc machine (either generator or motor) essentially consists of some stationary and some rotating parts. The basic construction of a dc machine is shown on the next page.



Stationary Parts :-

1) **Yoke or Frame :-** The outer frame of the dc machine is called the yoke and is normally made up of iron or fabricated steel or annealed steel laminations. It gives protection to inside rotating parts. It also provides return path for the magnetic flux produced by poles.

2) **Poles :-** The poles support the field winding which is mounted inside circumference of yoke. These produce magnetic flux. These are electromagnets. Coils over them are in series and produce alternate N and S polarity.

3) **Pole shoe :-** This is extended part of pole near rotating armature. Due to its enlarged area, more flux can pass through air gap upto armature. It also supports field coils over poles.

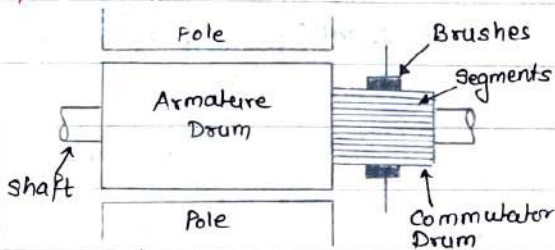
4) **Field winding :-** The field windings are mounted on the pole and produce alternate N and S polarity. DC supply is needed to excite these windings and to produce flux in poles. Also, since these windings are stationary, it is also called as stator winding. The field windings are generally made up of copper wire.

Rotating Parts :-

1) **Armature :-** An armature is a cylindrical drum mounted on the shaft. It is provided with large number of slots over the periphery in such a way that these slots are parallel to the shaft. It carries the flux from the poles.

2) **Armature Winding :-** This windings are placed on the slots of armature. These are made from copper. These cut the flux when armature is rotated. EMF is induced in them. Since it rotates along with armature, it is called as **rotor winding**.

3) **Commutator and Brushes :-** Commutator is a cylindrical drum mounted on the shaft along with the armature drum. The surface of the commutator is segmented and armature winding is tapped at various points on it. A set of

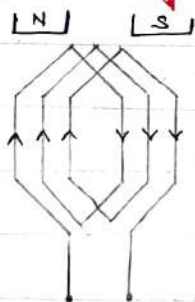


brushes rest on the surface of the commutator drum. The commutator segments collect the current through armature winding and pass through brushes. Brushes are made up of carbon.

* Types of Armature Windings :-

Depending on the manner in which the ends of the armature windings are connected to the commutator segments, there are two possible methods of connections: Lap winding and Wave winding.

Lap Winding :-



While connecting armature conductors, conductor under N pole is always connected to conductor under S pole to add two emf. In Lap connection this conductor is connected to next conductor under same N pole. Again it is connected to conductor under same S pole and so on.

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Thus all the conductors under one pole pair are connected. Then all the conductors under next pole pair are connected. Thus all conductors are covered with each pole-pair covered.

This method makes parallel groups of conductors. The number of parallel paths is number of poles.

ie

$$A = P$$

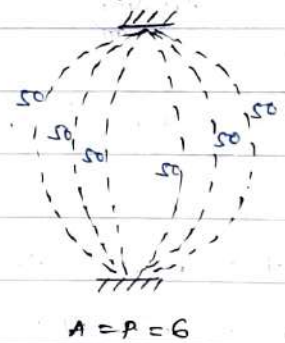
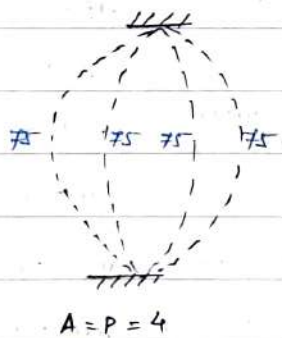
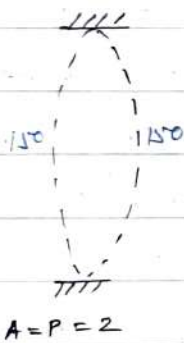
This connection is useful for high current but low voltage.

Eg:- If machine has 300 armature winding conductors, then

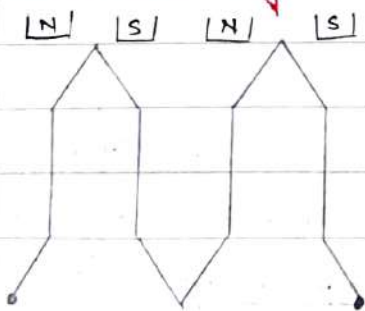
i) For 2-pole machine $\Rightarrow A = P = 2$ Conductors/pole = $\frac{300}{2} = 150$

ii) For 4-pole machine $\Rightarrow A = P = 4$ Conductors/pole = $\frac{300}{4} = 75$

iii) For 6 pole machine $\Rightarrow A = P = 6$ Conductors/pole = $\frac{300}{6} = 50$



Wave Winding :-



In this connection end of a coil is connected to next conductor under next similar pole. Hence in this case the conductors when connected come in series to form a single closed circuit. This closed circuit is tapped at various points.

This connection makes only two groups of the conductors in parallel with each other.

$\therefore$

$$A = 2$$

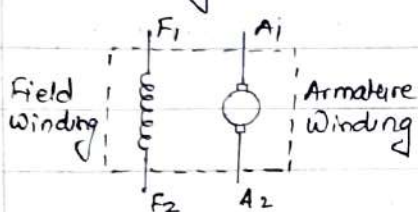
irrespective of number of poles.

This winding is used for low current-high voltage capacity machines.



Symbolic Representation of Machines:-

While representing the generator (or motor) the two windings of the generator viz. the Field (stator) winding and armature (rotor) winding are shown by two different symbols.



The Field winding is shown like a coil and its terminals are marked as F_1 & F_2 .

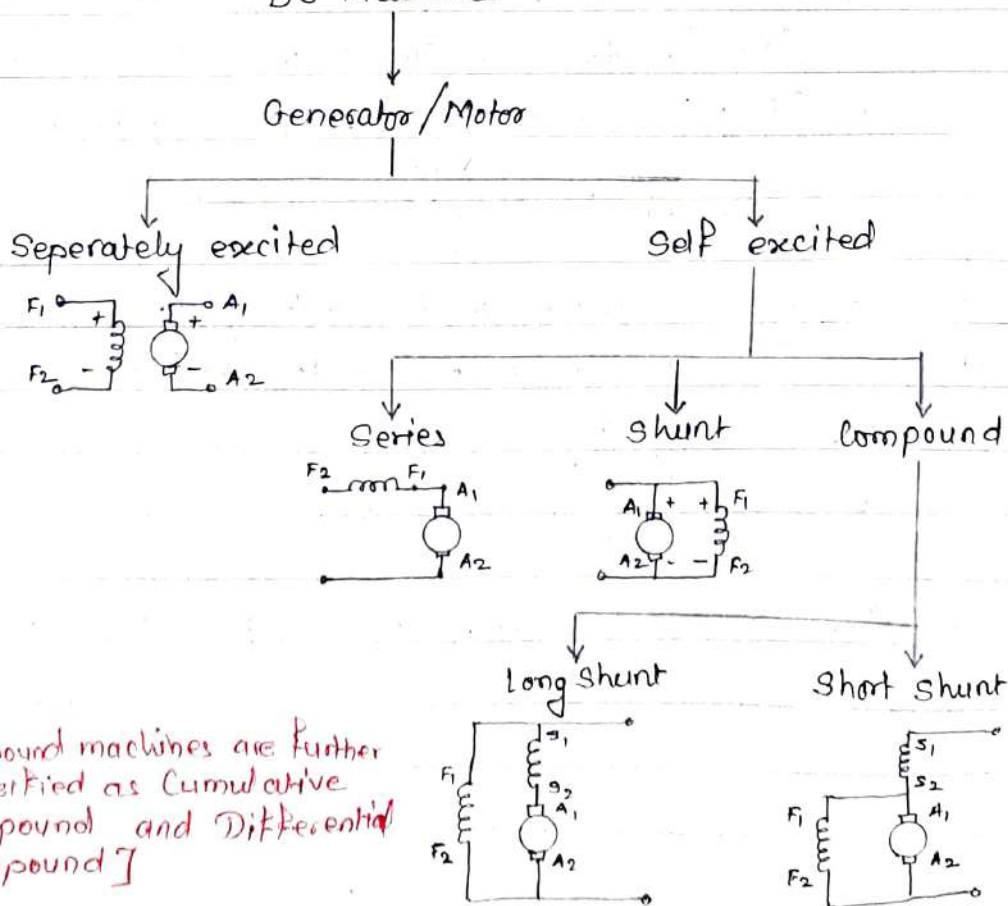
The armature winding is shown like a drum as it is placed on the armature drum. Its terminals are marked as

A_1 and A_2 . Two small projections are shown at the armature windings which indicates brushes.

* Types of DC Machines :-

There are different types of DC machines depending upon how we connect the field and the armature windings to each other and together to the supply

DC Machines



[Compound machines are further classified as Cumulative compound and Differential compound]

* EMF Equation of a DC Generator :- (motors)

A dc generator takes mechanical energy as input and delivers electrical energy as output. When a mechanical device, such as a diesel engine drives the armature drum, armature winding cuts the magnetic flux produced by the pole. An equation for emf induced can be obtained as under:

Let P = number of poles of the generator

ϕ = flux produced by each pole in Wb.

N = speed in rpm at which the generator is driven

Z = number of conductors of armature winding

A = number of parallel paths of armature winding

[note :- $A = P$ for lap winding
 $A = 2$ for wave winding]

Then according to Faraday's law of induction, magnitude of emf induced in a conductor is

$$E = \frac{d\phi}{dt} \quad \rightarrow m \rightarrow (1)$$

For one complete revolution of the conductor, the flux cut by the conductor is $(P \cdot \phi)$ webbers and time required to complete one revolution is $(60/N)$ seconds. Therefore eqn (1) can be written as

$$E = \frac{P \cdot \phi}{60/N} = \frac{P \phi N}{60} \quad \rightarrow m \rightarrow (2)$$

As Z conductors are divided in A parallel groups, there are Z/A conductors in series in each group. Therefore emf induced in the total armature winding is

$$E = \frac{P \phi N}{60} \cdot \frac{Z}{A} \quad \rightarrow m \rightarrow (3)$$

Equation (3) is called the emf equation of a dc generator.

* Frequency of the EMF induced in the Armature:-

The Frequency is defined as number of cycles completed in one second or reciprocal of time required to complete one cycle. If N is the speed in ~~amp~~ rpm at which machine is driven, then time required for one revolution = $\left(\frac{60}{N}\right)$ seconds.

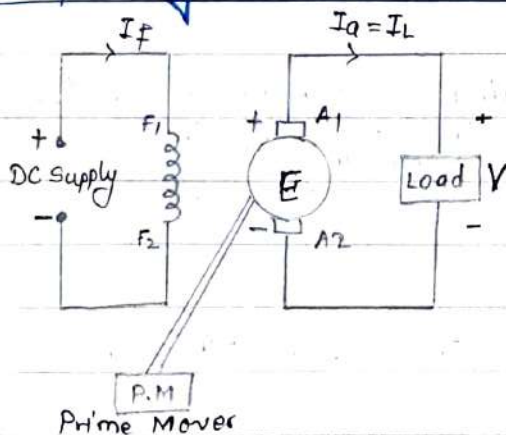
Number of cycles of emf induced in one revolution = $\left(\frac{P}{2}\right)$

$$\therefore \text{Time for one cycle} = \frac{60/N}{P/2} = \left(\frac{120}{P \times N}\right) \text{ seconds}$$

$$\therefore \text{Frequency of emf induced in the armature} = \frac{1}{T} = \left(\frac{P \times N}{120}\right) \text{ Hz}$$

Generators

1) Separately Excited Generator:-



In this case, Field winding gets its current from some other source. There is no connection between field winding and armature winding. The armature is driven by some engine called as Prime Mover. The field gets excited from separate source and armature

conductors cut these flux lines to develop output. This electrical energy is supplied to load.

Current Relations:-

Due to separate excitation

$$I_f = \text{Constant}$$

and $I_a = I_L$; since load is directly connected to armature

Voltage Relations:-

$$E = \frac{P \phi N}{60} \cdot \frac{Z}{A}$$

For a given generator, P , A and Z are constants. For this generator ϕ is also constant as I_f is constant.

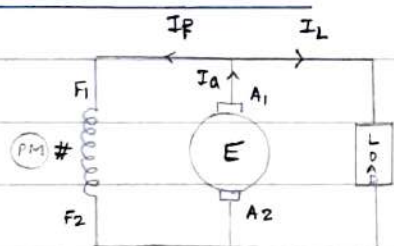
Hence, $E \propto N$. If speed of the prime mover is constant, induced emf will also be constant.

Terminal voltage

$$V = E - I_a R_a - \text{Brush drop.}$$

where $I_a R_a$ is the drop across resistance of armature winding.

2) Shunt Generator:-



Shunt generator is a self excited generator where field winding is fed by the armature itself. The two windings being connected in parallel as shown in Fig.

Current Relations:-

The armature winding is supplying the load as well as it is supplying its own field winding.

$$\therefore I_a = I_L + I_f$$

Voltage Relations:-

Emf induced in the armature winding is

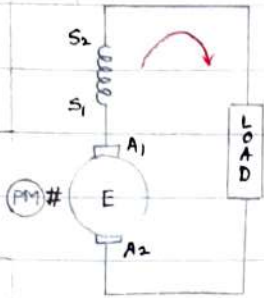
$$E = \frac{P \phi N}{60} \cdot \frac{Z}{A}$$

The voltage in the field winding (V_f), load voltage (V_L) and the voltage available at the armature terminals A_1 and A_2 (V_T) is same

$$\text{ie } V_L = V_f = V_T$$

$$\text{also } V_L = E - I_a R_a - \text{Brush drop.}$$

3) Series Generator :-



In series generator, the Field winding is connected in series with armature winding and this series combination supplies load. The Fig. shows such a generator. Here Field winding terminals are marked as s_1 & s_2 and not as f_1 & f_2 . just to indicate that is a series winding.

Current Relations :

The armature winding, the series Field winding and the load being in series.

$$I_L = I_a = I_s$$

Voltage Relations:-

$$E = \frac{P \Phi N}{60} \cdot \frac{Z}{A}$$

In case of series generator resistance of armature winding R_a as well as that of the field winding R_s , both create voltage drops.

$$\therefore V_L = E - I_a R_a - I_a R_s - \text{Brush drop}$$

Motor

* Significance of Back EMF:-

In previous discussion we have obtained emf induced for a generator. While deriving this equation it was noted that, the poles generate magnetic flux, this flux is cut by armature winding and according to Faraday's law of induction, emf is induced in the armature winding. When dc motor runs, the entire action noted above takes place and exactly in the similar way, emf is induced in the armature. Value of this emf is given as

$$E = \frac{P \Phi N}{60} \cdot \frac{Z}{A}$$

→ ①

This power is equal to $(E_b \cdot I_a)$ as per power equation.
∴ We can write

$$T \times \frac{2\pi N}{60} = E_b \cdot I_a$$

$$\therefore T \times \frac{2\pi N}{60} = \frac{P \phi N}{60} \cdot \frac{Z}{A} \cdot I_a$$

$$\therefore T = \frac{P \phi Z I_a}{2\pi A} = 0.159 \left(\frac{P \phi Z I_a}{A} \right) \text{ (N.m)}$$

The terms P , Z and A are constant for a motor and hence the torque can be varied by varying flux or armature current.

* Effect of Load on Motors:-

At no load motor draws very small current and power drawn is also less; since it has to overcome only the losses taking place in different windings and hence the speed of the motor is maximum.

As load is increased, more power is required to drive the load. Hence more current is drawn from the supply. This increases torque. As torque developed depends on the armature current. Correspondingly speed goes on decreasing. At very large loads, motor may stop rotating and is known as stalling conditions.

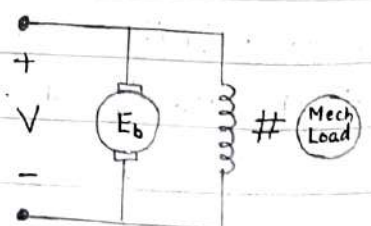
* Speed Control of DC Motors:-

Fig shows a dc motor a load. The voltage equation of the motor is

$$V - I_a R_a = E_b \quad \rightarrow (1)$$

$$\text{where } E_b = \frac{P \phi N}{60} \cdot \frac{Z}{A} \quad \rightarrow (2)$$

$$\text{or } E_b \propto \phi N \quad \rightarrow (3)$$



In practice, if supply voltage V is approximately equal to back emf because $I_a R_a$ drop is very small.

ie $V \approx E_b$

Substituting in eqn (3)

$$V \propto \phi N \quad \text{ie} \quad N \propto \frac{V}{\phi} \quad \text{--- (4)}$$

Equation (4) indicates that the speed of dc motor can be controlled in two ways:

- i) by controlling motor supply voltage V
- ii) by controlling magnetic flux

* Necessity of starter:-

For a simple dc motor, the armature current is given as

$$I_a = \frac{V - E_b}{R_a}$$

where V is the applied voltage and R_a is armature winding resistance.

In practice, at the instant of starting ie when supply is switched on, the speed N of the motor is zero and hence Back emf E_b is also zero. Hence armature current-

$$I_a = \frac{V - 0}{R_a} = \frac{V}{R_a}$$

Eg: $V = 230, E_b = 225V, R_a = 1\Omega$
 $I_{a \text{ start}} = \frac{230 - 0}{1} = 230A$
 $I_{a \text{ normal}} = \frac{230 - 225}{1} = 5A$

which is very high.

During normal operation, motor will rotate at its rated speed and Back emf E_b is approximately equal to V . Hence in this case armature current is very small.

Hence, for dc motor starting current is very high and this high current may damage armature winding instantly. This high current can also damage supply. To protect the motor from such dangerous current a protective device

called as starter is employed.

starter is a external resistance which is connected in series with the armature during starting and as motor takes up speed E_b increases. Gradually the external resistance is decreased and made zero when the motor attains rated speed.

* Applications of D.C. Motors:-

① Shunt Motor applications:-

- Various machine tools such as Lathe machines, drilling machines, milling machines etc.
- Printing machinery
- Paper machines
- Centrifugal and reciprocating pumps
- Blowers and fans etc.

② Series Motor applications:-

- Electric trains
- Diesel-electric locomotives
- Cranes
- Hoists
- Trolley cars and trolley buses
- Rapid-transit systems
- Conveyers etc.

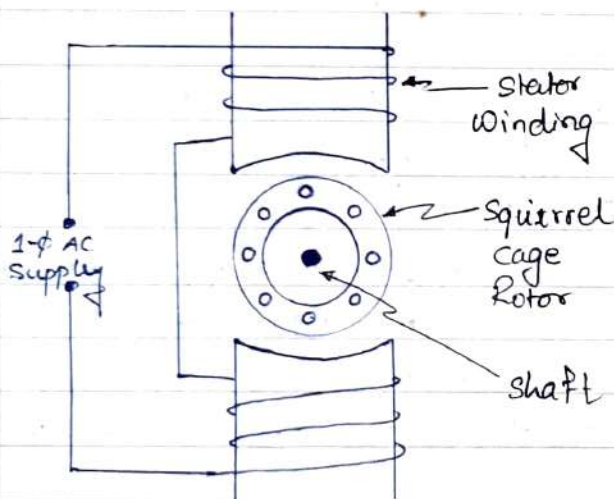
Single Phase Induction Motor

The mains supply available at homes, shops, offices, schools etc. is a single phase ac supply. Hence instead of dc motors, the motors which work on single phase ac supply is more suitable. These ac motors are called single phase induction motors.

* Construction →

Similar to dc motor, ac motor (single phase induction motor) has basically two main parts, one rotating and other stationary. The stationary part here is called **stator**, while the rotating part is called **rotor**.

The stator has a

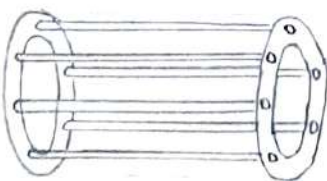


laminated construction, made up of stampings. The stampings are slotted on the periphery to carry **stator winding**. This is excited by a single phase ac supply. The laminated construction keeps iron losses minimum. The stamping minimises

hysteresis loss. The number of poles decide the speed of the motor. This speed is called **Synchronous Speed (N_s)** and is given by

$$N_s = \frac{120 f}{P}$$

where f = Freq. of ac supply
 P = No. of poles



The rotor construction is of **squirrel cage type**. In this type, rotor consists of unisulated copper or aluminium bars, placed in the slots. The bars are permanently shorted at both ends with the help of a conducting ring. The entire structure looks like cage, hence called **squirrel cage rotor**.

* Working →

* Double Revolving Field theory

* Why single phase Induction motor is not self starting

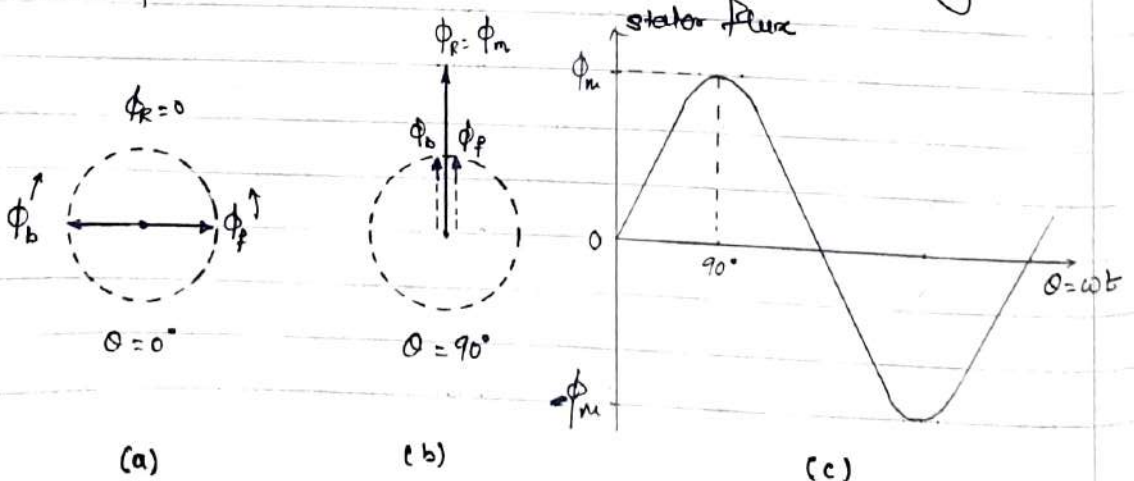
According to this theory, any alternating quantity can be resolved into two rotating components which rotate in opposite directions, and each having magnitude as half of the maximum magnitude of the alternating quantity.

$$\text{Note} \rightarrow \phi_m \sin \omega t = \phi_m \left[\frac{e^{j\omega t} - e^{-j\omega t}}{2j} \right]$$

Here $e^{-j\omega t}$ component rotates in clockwise direction and $e^{j\omega t}$ rotates in anti-clockwise direction, each having magnitude of $\phi_m/2$.

In the case of single phase induction motors, the stator winding produces an alternating magnetic field having maximum magnitude of ϕ_m . According to double revolving field theory, it can be resolved in two rotating components each with vector length of $\phi_m/2$ rotating in opposite directions. The speed of these rotating vectors is equal to the synchronous speed N_s which depends on frequency and stator poles.

Let ϕ_f be forward (anticlockwise) rotating component and ϕ_b be backward (clockwise) rotating component. The resultant of these two components at any instant gives the instantaneous value of the stator flux at that instant. Hence, resultant of these two is the original stator flux.



Note → If Double Revolving Field theory is for 8 Marks also add torque-speed characteristics

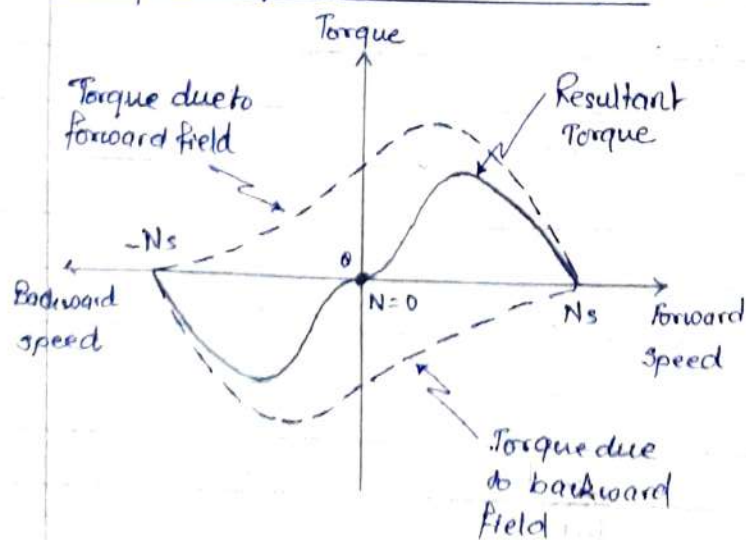
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Fig above shows the stator flux and its two components ϕ_f and ϕ_b . At the start both the components are shown opposite to each other [see Fig (9)]. Thus the resultant $\phi_r = 0$. This is nothing but the instantaneous flux at start. After 90° , as shown in Fig (b), the two components are pointing in the same direction. Hence the resultant $\phi_r = \phi_m/2 + \phi_m/2 = \phi_m$. This is nothing but the instantaneous value of the stator flux at $\theta = 90^\circ$. Thus continuous rotation of the two components gives the original alternating stator flux as shown in Fig (c).

Both the components are rotating and hence get cut by the rotor conductors. Due to cutting flux emf gets induced in rotor which circulates rotor current. The rotor current produces rotor flux. This flux interacts with forward component ϕ_f to produce a torque in one particular direction say anticlockwise direction. While rotor flux interacts with backward component ϕ_b to produce a torque in the clockwise direction. So if anticlockwise torque is positive then clockwise torque will be negative.

At the start, these two components are equal in magnitude but opposite in direction. Hence the net torque is zero at the start. Hence, the rotor will not move due to equal and opposite torques acting on it. Hence, single phase induction motors are not self-starting.

* Torque - Speed Characteristics →



The two oppositely directed torques and the resultant torque can be shown effectively by torque-speed characteristics.

It can be seen that, at start $N=0$ and at that point resultant

torque is zero. Hence, single phase induction motors are not self starting.

However, if the rotor is given an initial rotation in any direction, the resultant average torque increases in the direction in which rotor is initially rotated. And motor starts rotating in that direction.

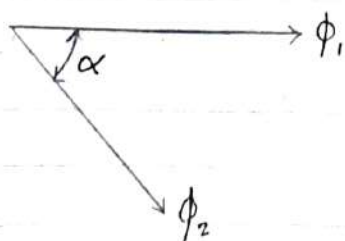
But in practice it is not possible to give initial torque to the rotor externally, hence some modifications are done in the construction of single phase induction motors to make them self starting.

* Starting Methods / Split Phase Technique →

In case of single phase induction motors, some constructional arrangement is required to make the motors self starting.

In case of split phase technique the arrangement is made so that the stator produces a rotating flux rather than alternating flux, which rotates in one particular direction only. Hence, torque produced due to such magnetic field is unidirectional as there is no oppositely directed torque present. Hence under the influence of rotating magnetic field in one direction, the induction motor becomes self starting.

In case of induction motors, to convert the alternating flux to rotating flux, we produce an additional flux other than stator flux which has a certain phase difference w.r.t. stator flux. Hence the resultant is a rotating flux rather than alternating flux. More the

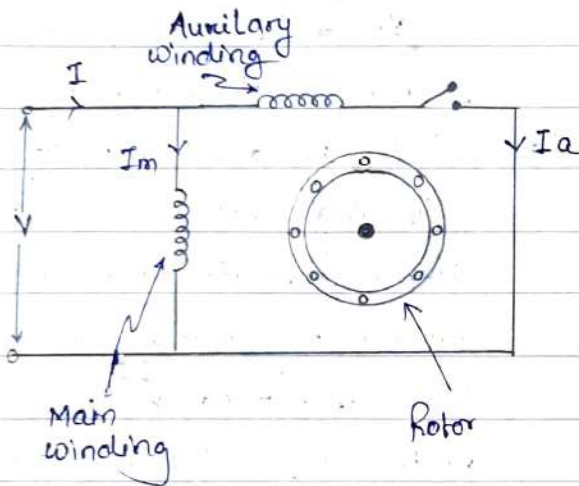


phase splitting angle α , more is the starting torque. This additional flux is produced by auxiliary winding placed along with stator winding. The auxiliary winding is then removed once motor starts, by a switch arrangement.

There are ~~three~~ types of split-phase induction motor.

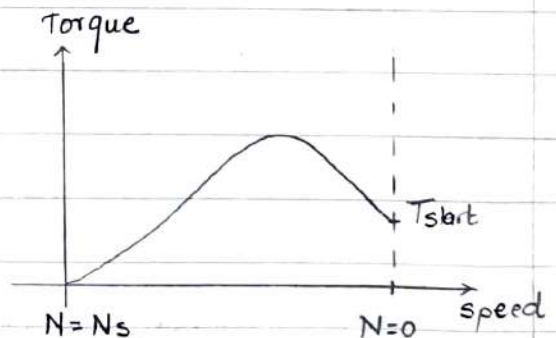
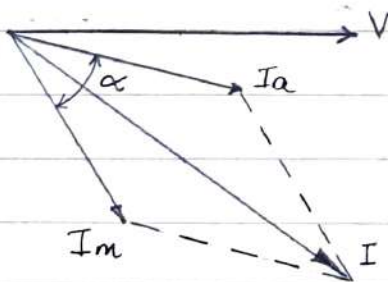
- i) Resistance split phase motor
- ii) Capacitor start motor
- iii) Capacitor start capacitor run motor

① Resistance Split-phase motor →



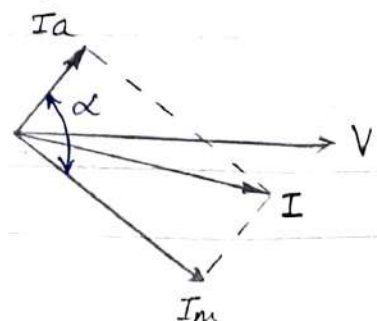
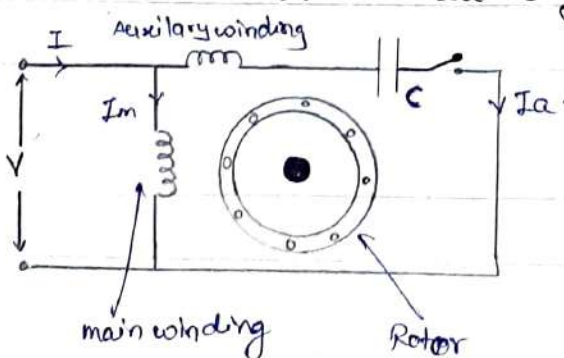
Here the auxiliary winding is made up of a thin copper conductor as compared to main winding. Hence auxiliary winding has much higher resistance compared to inductive main winding. Hence the fluxes will be split inphase and motor becomes self starting. The phasor diagram and torque speed characteristics are as shown below.

self starting. The phasor diagram and torque speed characteristics are as shown below.



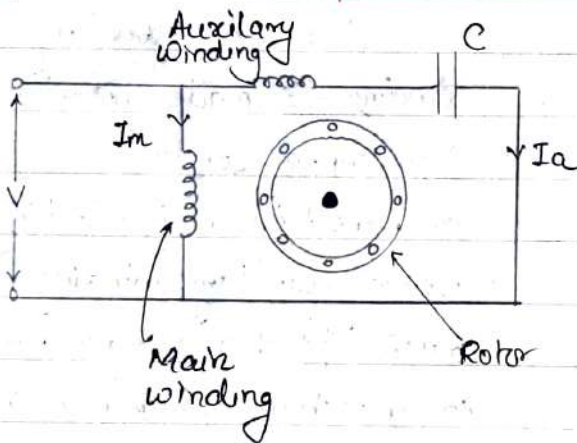
② Capacitor start motor →

In this case a condensor (capacitor) is placed in series with the auxiliary winding.

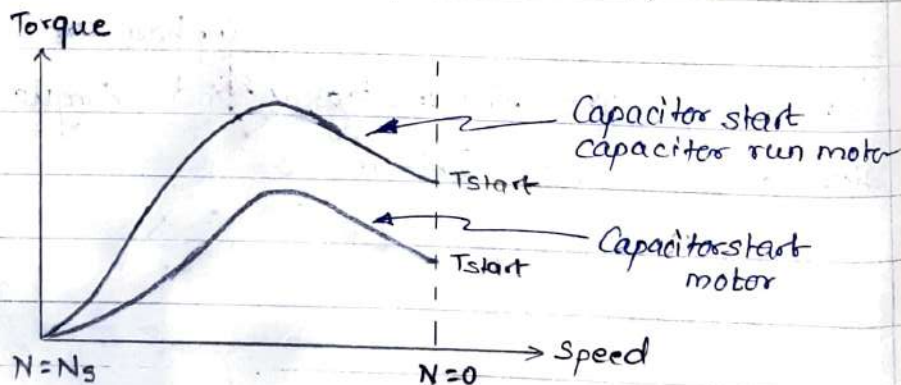


Due to capacitor C , current in auxiliary winding is leading voltage V , and current in main winding is lagging. Hence splitting angle α is more, which produces higher starting torque.

⑤ Capacitor start - Capacitor Run motor $\rightarrow$



In this case, there is no switch present which removes auxiliary winding once motor starts. Hence capacitor remains permanently in the circuit. This improves overall performance of the motor.



* Applications $\rightarrow$

The split-phase induction motors are used in variety of common applications like fans, coolers, pumps, air conditioners etc.

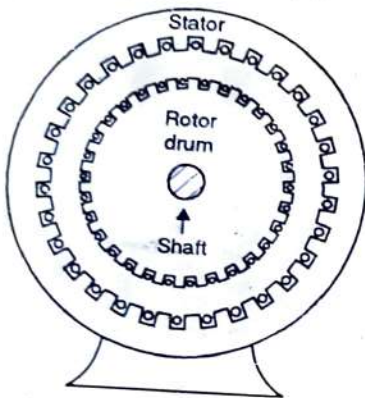
3- ϕ INDUCTION MOTOR

(21)

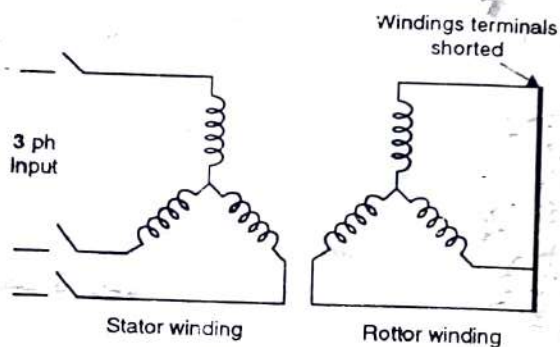
* Construction :-

Basically induction motor consists of two main parts

- the stationary frame called as **stator**, and
- the rotating armature called as **Rotor**.



(a)



(b)

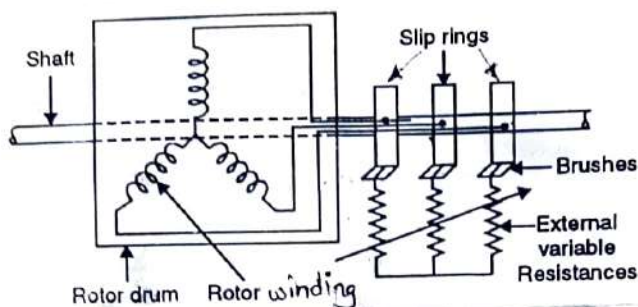
Fig (a) shows the schematic diagram of the induction motor. The stator of an induction motor, in principle, is the same as that of a synchronous motor or generator. It is made up of a number of stampings, which are slotted to receive the windings. The stator carries a 3ϕ winding and is fed with a 3ϕ supply. It is wound for a definite number of poles, the exact number of poles is determined by the requirement of the speed. Greater the number of poles, lesser the speed and vice versa.

Similar to stator, the rotor drum is provided with slots and those slots carry a three phase winding similar to stator winding. The two windings are redrawn in Fig. (b). (Note $\rightarrow$ windings can be connected in star as well as delta). The winding terminals of rotor winding are shorted together.

Based on the construction of the rotor we have two types, viz. slip-ring rotor and squirrel cage rotor.

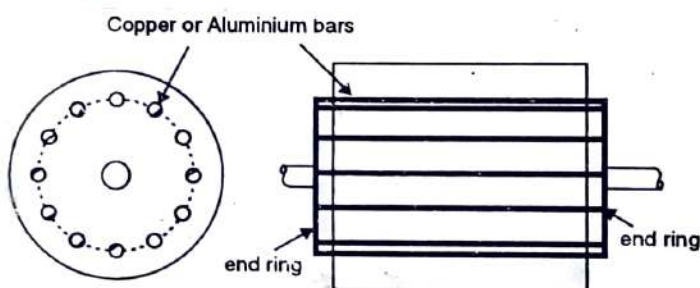
• Slip-Ring Rotor :-

In this type of rotor, along with the rotor drum the shaft carries three slip-rings. The winding terminals are permanently connected to these rings. Many times it is necessary to add external resistance in the rotor



circuit. Such addition is necessary if the motor is required to develop higher starting torque. Thus the external resistance can be added through slip rings-brushes arrangement.

• Squirrel Cage Rotor :-



When higher starting torque is not the requirement, the rotor is constructed in a very simple manner. Fig above shows the construction of a squirrel cage rotor. Here the rotor drum is provided with a number of circular holes, parallel to the shaft. Solid copper or aluminium bars are placed in these holes and all these bars are welded to two end rings to establish electrical contact between them. In the Fig. above, the ring is shown with dotted line for clarity. The bars and end rings together look like a cage and hence such rotor is called as cage rotor or squirrel cage rotor.

Slip - Ring Rotor	Squirrel - cage rotor
• Rotor construction is complicated as the rotor drum is to be provided with slots and rotor windings is to be placed properly.	• Construction is simple as the winding is nothing but solid bars placed in holes.

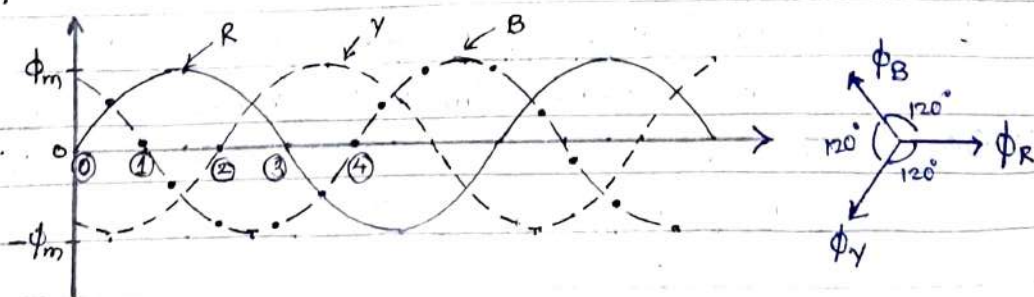
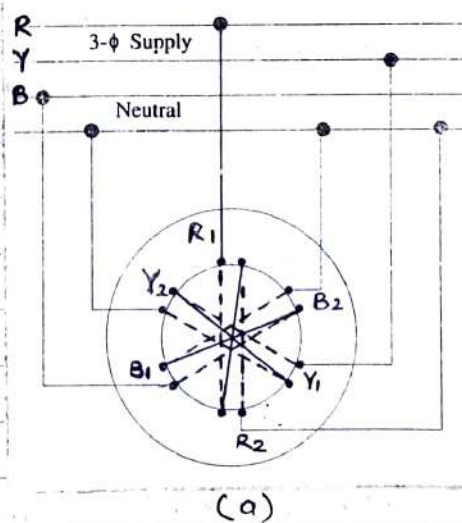
- | | |
|---|---|
| <ul style="list-style-type: none"> • Produces high starting torque. • Cost of slip rotor is high. | <ul style="list-style-type: none"> • Produces low starting torque. • Cage rotor comparatively, is cheaper in cost. |
| <ul style="list-style-type: none"> • Only 5% of induction motor used in industry employ the slip ring rotor. | <ul style="list-style-type: none"> • Nearly 95% induction motors employ cage rotors. |
| <ul style="list-style-type: none"> • Extra resistance can be added in rotor circuit to enable the motor to develop higher starting torque. | <ul style="list-style-type: none"> • Extra resistance cannot be added in rotor circuit, therefore, if the motor is required to develop high starting torque, cage rotor can not be used. |

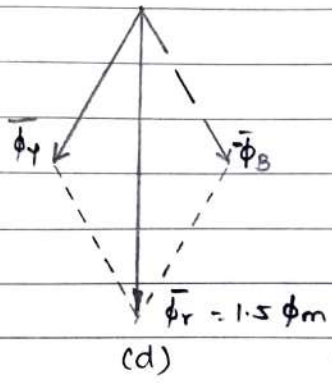
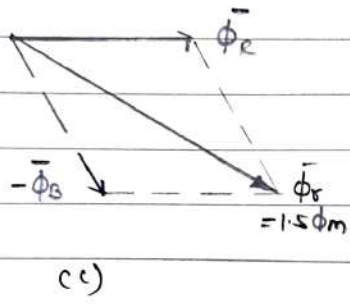
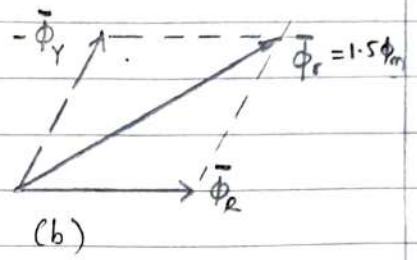
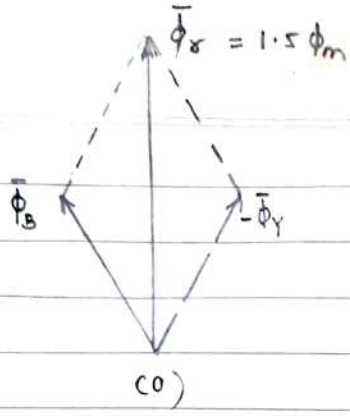
* Working :- (Rotating Magnetic Field)

In 3 ϕ induction motor, when the stator windings are fed by 3 ϕ currents displaced by 120° , they produce a resultant magnetic flux, which rotates in space as if actual magnetic poles are being rotated mechanically.

The principle of 3 ϕ , 1 pole stator having three identical winding is illustrated in Fig. (a). When excited with 3 ϕ supply, the flux due to the current flowing in each phase winding will be sinusoidal and is represented in Fig. (b). The maximum value of flux is ϕ_m . The

resultant flux ϕ_R at any given time is given by the vector sum of the individual fluxes ϕ_R , ϕ_Y and ϕ_B due to three phases.





at point ① i.e. when $\theta = 0^\circ \Rightarrow$ Fig (a)

$$\phi_R = 0, \quad \phi_Y = -\frac{\sqrt{3}}{2}\phi_m, \quad \phi_B = \frac{\sqrt{3}}{2}\phi_m$$

$$\therefore \phi_R = \vec{\phi}_R + \vec{\phi}_Y + \vec{\phi}_B = \frac{\sqrt{3}}{2}\phi_m \times \frac{\sqrt{3}}{2}\phi_m \times \cos\left(\frac{60^\circ}{2}\right) = 1.5\phi_m$$

at point ② i.e. when $\theta = 60^\circ \Rightarrow$ Fig (b)

$$\phi_R = \frac{\sqrt{3}}{2}\phi_m, \quad \phi_Y = -\frac{\sqrt{3}}{2}\phi_m, \quad \phi_B = 0$$

$$\therefore \phi_R = \frac{\sqrt{3}}{2}\phi_m \times \frac{\sqrt{3}}{2}\phi_m \times \cos\left(\frac{60^\circ}{2}\right) = 1.5\phi_m$$

Here resultant is $1.5\phi_m$ but rotated by 60° in clockwise direction.

at point ③ i.e. when $\theta = 120^\circ \Rightarrow$ Fig (c)

$$\phi_R = \frac{\sqrt{3}}{2}\phi_m, \quad \phi_Y = 0, \quad \phi_B = -\frac{\sqrt{3}}{2}\phi_m$$

$$\therefore \phi_R = \frac{\sqrt{3}}{2}\phi_m \times \frac{\sqrt{3}}{2}\phi_m \times \cos\left(\frac{60^\circ}{2}\right) = 1.5\phi_m$$

Again the resultant is same i.e. $1.5\phi_m$ but again rotated by 60° in clockwise direction.

at point (3) i.e. at $\omega = 180^\circ \Rightarrow$ Fig (d)

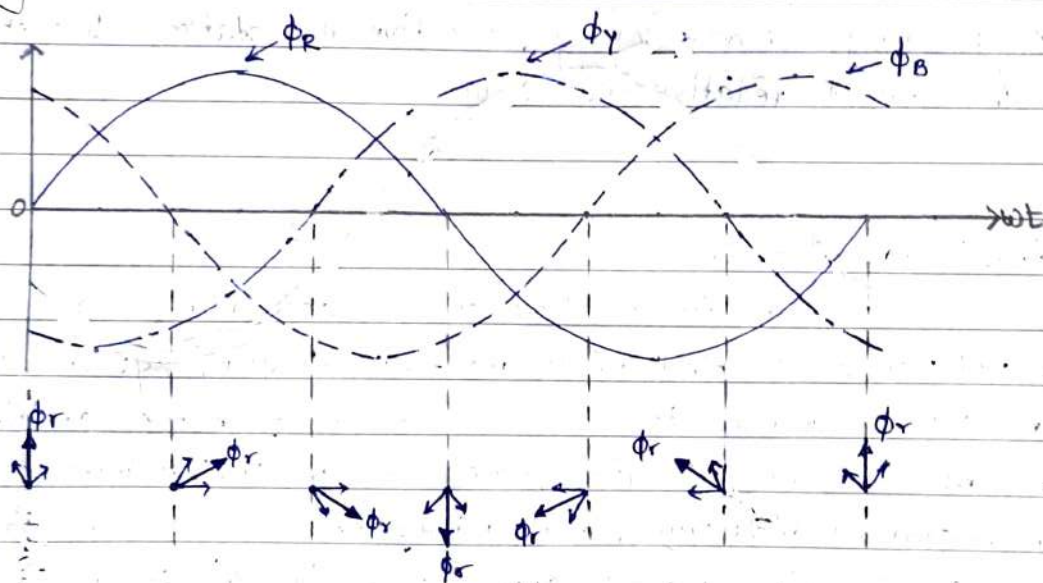
$$\phi_R = 0, \quad \phi_Y = \frac{\sqrt{3}}{2} \phi_m, \quad \phi_B = -\frac{\sqrt{3}}{2} \phi_m$$

$$\therefore \phi_r = \frac{\sqrt{3}}{2} \phi_m \times \frac{\sqrt{3}}{2} \phi_m \times \cos\left(\frac{60^\circ}{2}\right)$$

$$= 1.5 \phi_m$$

Again the resultant is same with an additional clockwise rotation of 60° .

Fig below show the graph of the rotating flux in a simple way.



Hence we can conclude that:

- (i) the resultant flux is of constant magnitude i.e. $1.5 \phi_m$
- (ii) the resultant flux rotates around the stator at synchronous speed given by $N_s = \frac{120f}{p}$

* Why does rotor rotate?

When the 3 ϕ stator windings are fed by a 3 ϕ supply, it sets up a magnetic flux of constant magnitude but rotating at synchronous speed. This induces emf in the rotor windings by Faraday's law of electro-magnetic induction. As the flux rotates, it also rotates the rotor to keep the relative speed same thus causing the rotation of the rotor.

* Slip :-

The difference between synchronous speed N_s and the actual speed N of the rotor is known as slip. It is expressed as the percentage of the synchronous speed. Actually, the term slip is descriptive of the way in which the rotor 'slips back' from synchronism.

$$\% \text{ slip} = s = \frac{N_s - N}{N_s} \times 100$$

Sometimes $N_s - N$ is called slip speed.

$\therefore$ Rotor (or motor) speed $N = N_s(1-s)$

It may be kept in mind that revolving flux is rotating synchronously, relative to stator, but at slip speed relative to rotor.

* Applications :-

- Due to constant speed and minimum maintenance coupled with moderate torque, squirrel cage motors are used to drive reciprocating pumps, centrifugal pumps, latches, to drive generators, in textile industry, in chemical industry, coal mines etc.

- Slip ring motors offer higher starting torque and controllable speed. Hence it is used in electric traction, elevators, lifts, cranes etc.